

Lecture 10

2022/2023

Microwave Devices and Circuits for Radiocommunications

2022/2023

- 2C/1L, **MDCR**
- Attendance at minimum 7 sessions (course or laboratory)
- Lectures- **associate professor Radu Damian**
 - Tuesday 12-14, ~~Online~~, P8
 - E – 50% final grade
 - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
 - first test L1: 21-28.02.2023 (t₂ and t₃ not announced, lecture)
 - 3att.=+0.5p
 - all materials/equipments authorized

2022/2023

- Laboratory – **associate professor Radu Damian**
 - Tuesday 08-12, II.13 / (08:10)
 - L – 25% final grade
 - ADS, 4 sessions
 - Attendance + **personal results**
 - P – 25% final grade
 - ADS, 3 sessions (-1? 21.02.2022)
 - personal homework

Materials

■ <http://rf-opto.etti.tuiasi.ro>

The screenshot shows a web browser displaying the website http://rf-opto.etti.tuiasi.ro/microwave_cd.php?ch_lang=0. The page title is "Microwave Devices and Circuits for Radiocommunications (English)". The main content area includes sections for Course (MDCR 2017-2018), Activities, Evaluation, Grades, Attendance, Lists, and Materials. The right side features the RF-OPTO logo, a globe graphic, and language links (English, Romana). A red circle highlights the "English" link in the language bar.

Laboratorul de Microunde și Optică

Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, Sem. 7

Activities

Evaluation

Type: Examen

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

[Aggregate Results](#)

Attendance

[Course](#)
[Laboratory](#)

Lists

[Bonus-uri acumulate \(final\)](#)
[Studentii care nu pot intra in examen](#)

Materials

Course Slides

MDCR Lecture_1 (pdf, 5.43 MB, en,)
MDCR Lecture_2 (pdf, 3.67 MB, en,)
MDCR Lecture_3 (pdf, 4.76 MB, en,)
MDCR Lecture_4 (pdf, 5.58 MB, en,)

ETII

RF-OPTO

UNIVERSITATEA TEHNICĂ DIN TÎRGU JIU
1813 LASI

English | Romana |

Main Courses Master Staff Research

Grades Student List Exams Photos

Online Exams

In order to participate at online exams you must get ready following

Materials

- RF-OPTO
 - <http://rf-opto.eti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**
Wiley; 4th edition , 2011
 - 1 exam problem ← Pozar
- Photos
 - sent by **email**/online exam
 - used at lectures/laboratory

Access

- Not customized

A screenshot of a student profile page. On the left is a thumbnail photo of a student. Below it is a link "Acceseaza ca acest student". To the right is a section titled "Date:" containing the following information:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Below this is a section titled "Note obtinute" with a table:

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

A screenshot of a contact form. It includes fields for "Nume" (Name) with a redacted value, "Email" (Email), and "Cod de verificare" (Verification code) with a redacted value. A large red arrow points from the "Email" field on the left to the "Email" field on the right. At the bottom is a button labeled "Trimite" (Send).

344bd9f

Online

- access to **online exams** requires the **password** received by email

English | Romana |

Main Courses Master Staff Research **Student List**

Grades Student List Exams Photos

POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telecomunicatii
Marca	7000000

[Access the site as this student](#) | [Request access to software](#)

Grades

Inca nu a fost notat.

Main Courses Master Staff Research

Grades **Student List** Exams Photos

Login

Use the last name and email stored in the database

Name
POPESCU GOPO

Email/Password

Write the code below

828f26b

Send

Online

- access email/password

Main Courses Master Staff Research

Grades Student List Exams Photos

POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student!**

Main Courses Master Staff Research

Grades Student List Exams Photos

POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telec
Marca	7000000

You access the site as **this student (including exams)!**

Password

■ received by email

Important message from RF-OPTO Inbox x

Radu-Florin Damian
to me, POPESCU ▾

Romanian ▾ English ▾ Translate message

 Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is
Password: [REDACTED]

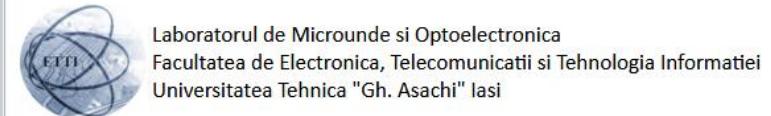
Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject: Important message from RF-OPTO Correspondents: POPESCU GOPO ION

From: Me <rdamian@etti.tuiasi.ro> ★
Subject: Important message from RF-OPTO (highlighted)
To: [REDACTED]
Cc: Me <rdamian@etti.tuiasi.ro> ★



In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul rf-opto este
Parola: [REDACTED]

Identificati-vă pe [server](#), cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

Attention: POPESCU GOPO ION

The password to access the exams on the rf-opto server is
Password: [REDACTED]

Login to the [server](#), with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Online exam manual

- The online exam app used for:
 - ~~lectures (attendance)~~
 - laboratory
 - project
 - ~~examinations~~

Materials

Other data

[Manual examen on-line \(pdf, 2.65 MB, ro, !\[\]\(8b57f0e15e7dda24cf9977561475f640_img.jpg\)](#)

[Simulare Examen \(video\) \(mp4, 65.12 MB, ro, !\[\]\(4cafc60cd39da821525d7c6589540296_img.jpg\)](#)

Examen online

- always against a **timetable**
 - long period (lecture attendance/laboratory results)
 - ~~short period (tests: 15min, exam: 2h)~~

Announcement 23:59 (10/05/2020)	Support material 00:05 (11/05/2020)	Exam Topics 00:07 (11/05/2020)	Results 00:10 (11/05/2020)	End 00:20 (15/05/2020)	Confirmation 00:20 (16/05/2020)	Next timeframe in: 05 m 43 s Refresh now
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Announcement

This is a "fake" exam, introduced to familiarize you with the server interface and to perform the necessary actions during an exam: thesis scan, selfie, use email for co...

Server Time

All exams are based on the server's time zone (it may be different from local time). For reference time on the server is now:

10/05/2020 23:59:16

Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgromot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri S	T2, fisier parmetri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

i	Z1	Z2	Z3	Z4	Z5	Z6	Z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10,693
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



Online results submission

Grade = Quality of the work +
+ Quality of the submission

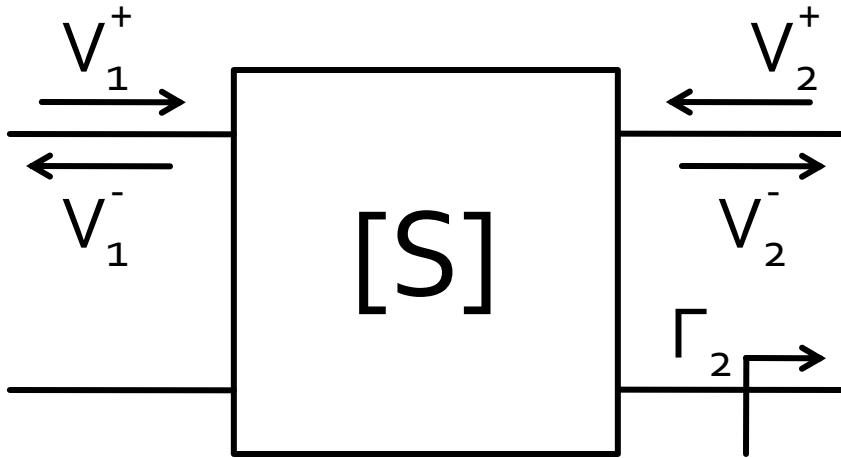
Recap

General theory

Microwave Network Analysis

Scattering matrix – S

■ Scattering parameters



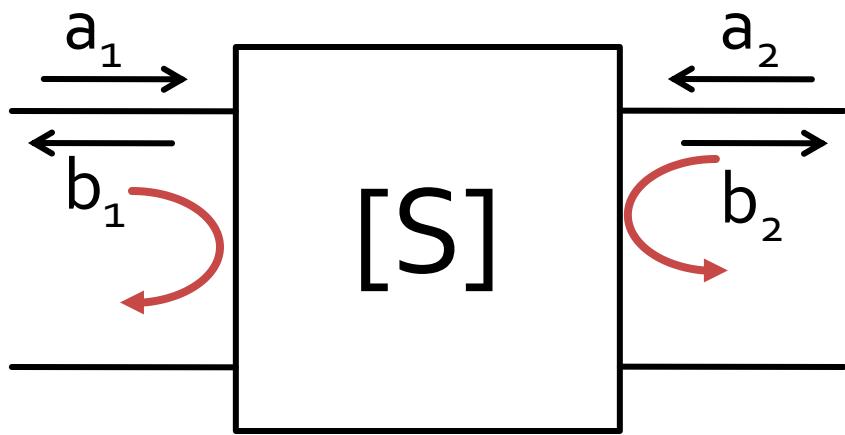
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} \quad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S

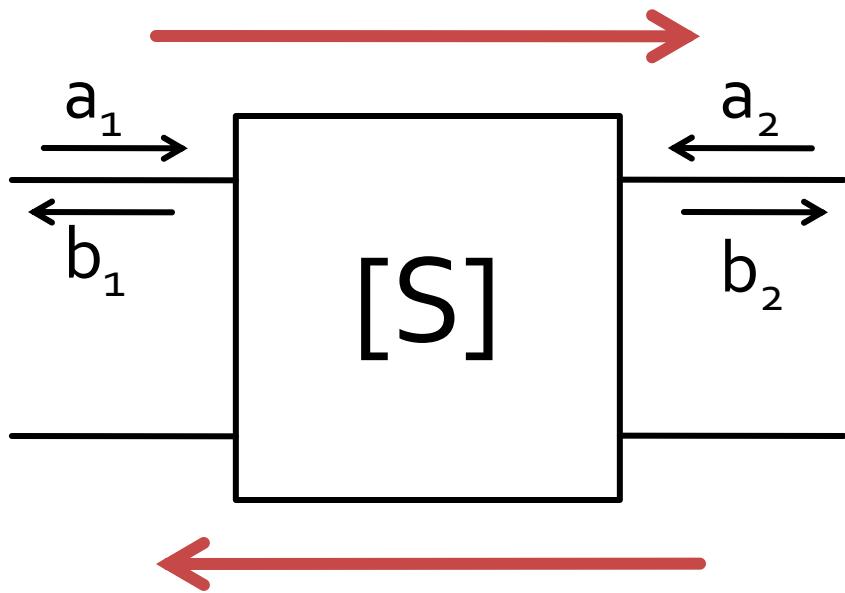


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \frac{b_1}{a_1} \Big|_{a_2=0} \quad S_{22} = \frac{b_2}{a_2} \Big|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is matched

Scattering matrix – S



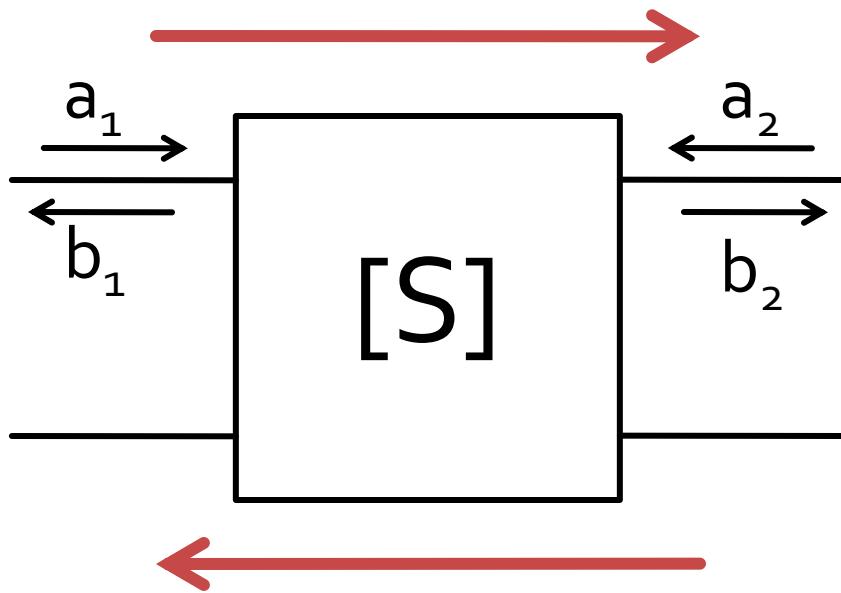
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \frac{b_2}{a_1} \Big|_{a_2=0}$$

$$S_{12} = \frac{b_1}{a_2} \Big|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

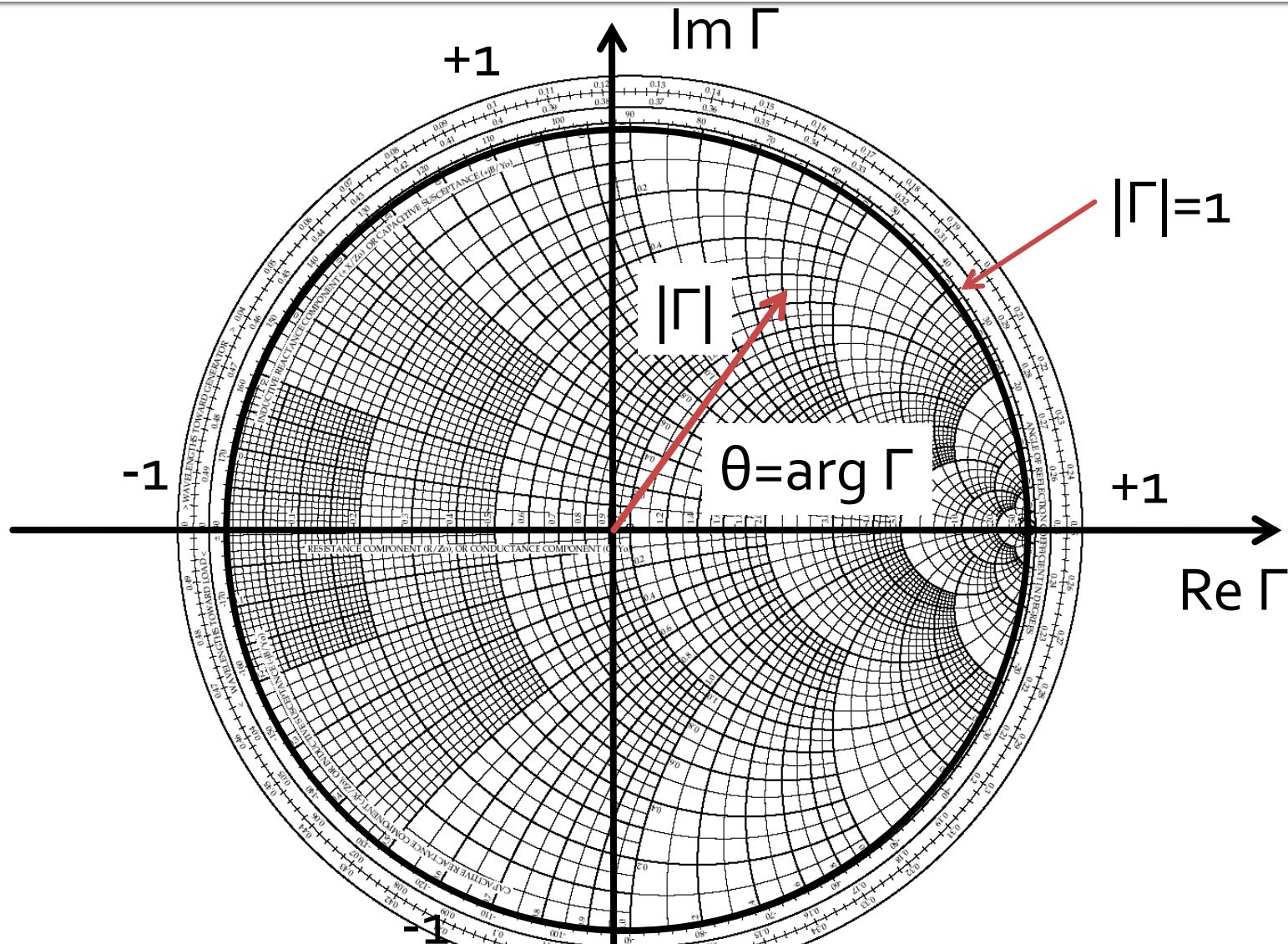
$$|S_{21}|^2 = \frac{\text{Power in } Z_0 \text{ load}}{\text{Power from } Z_0 \text{ source}}$$

- a, b
 - information about signal power **AND** signal phase
- S_{ij}
 - network effect (gain) over signal power **including** phase information

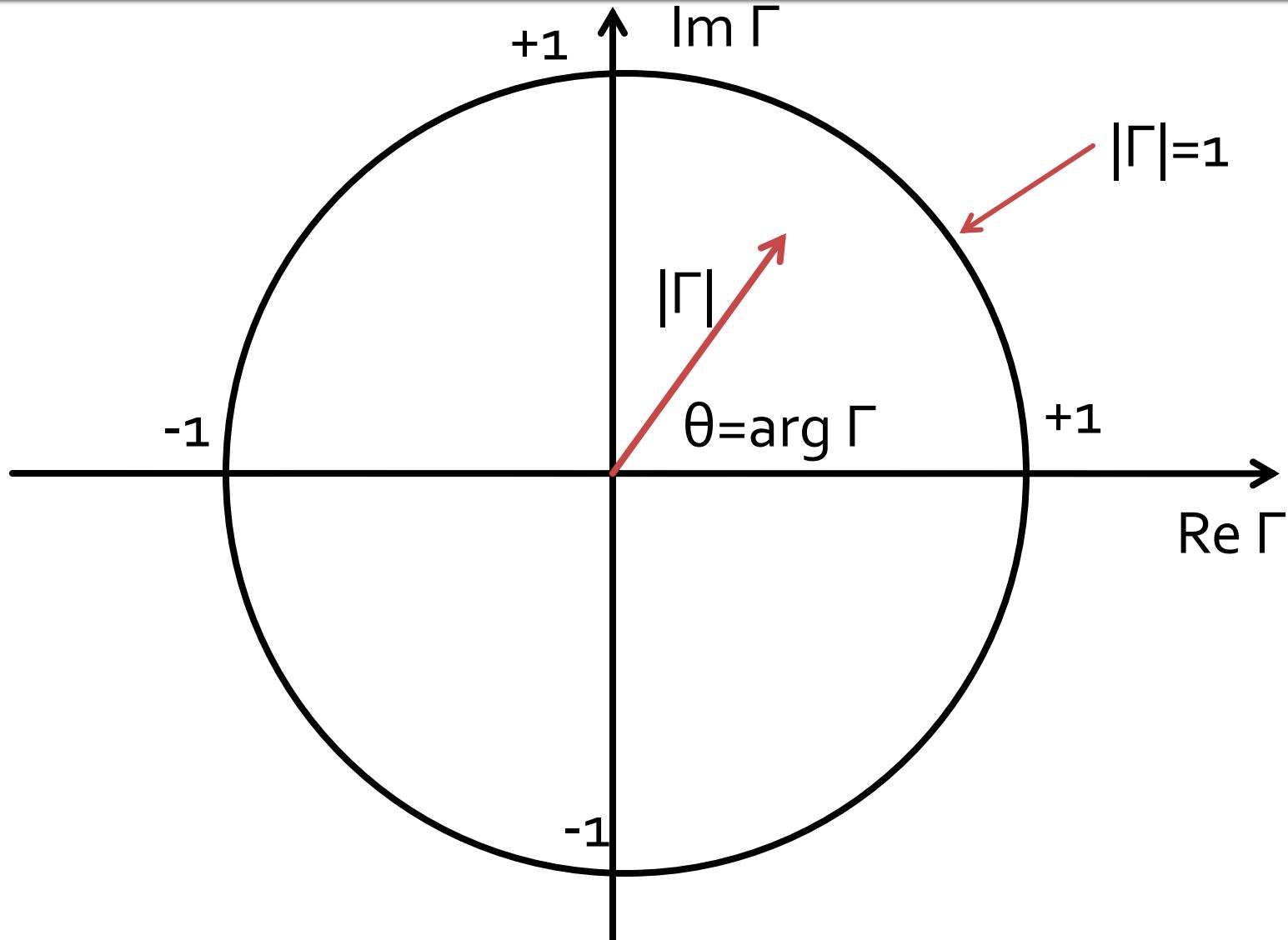
Impedance Matching

The Smith Chart

The Smith Chart



The Smith Chart



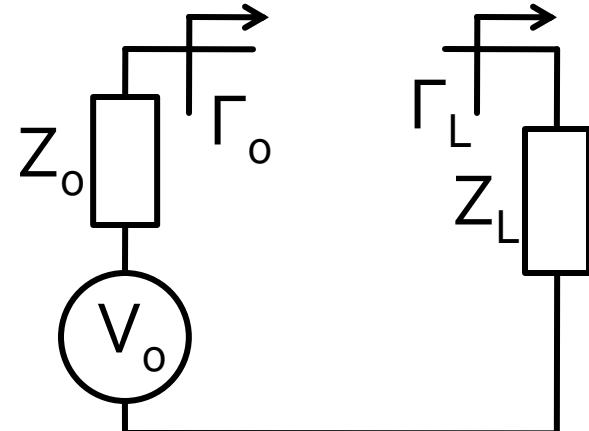
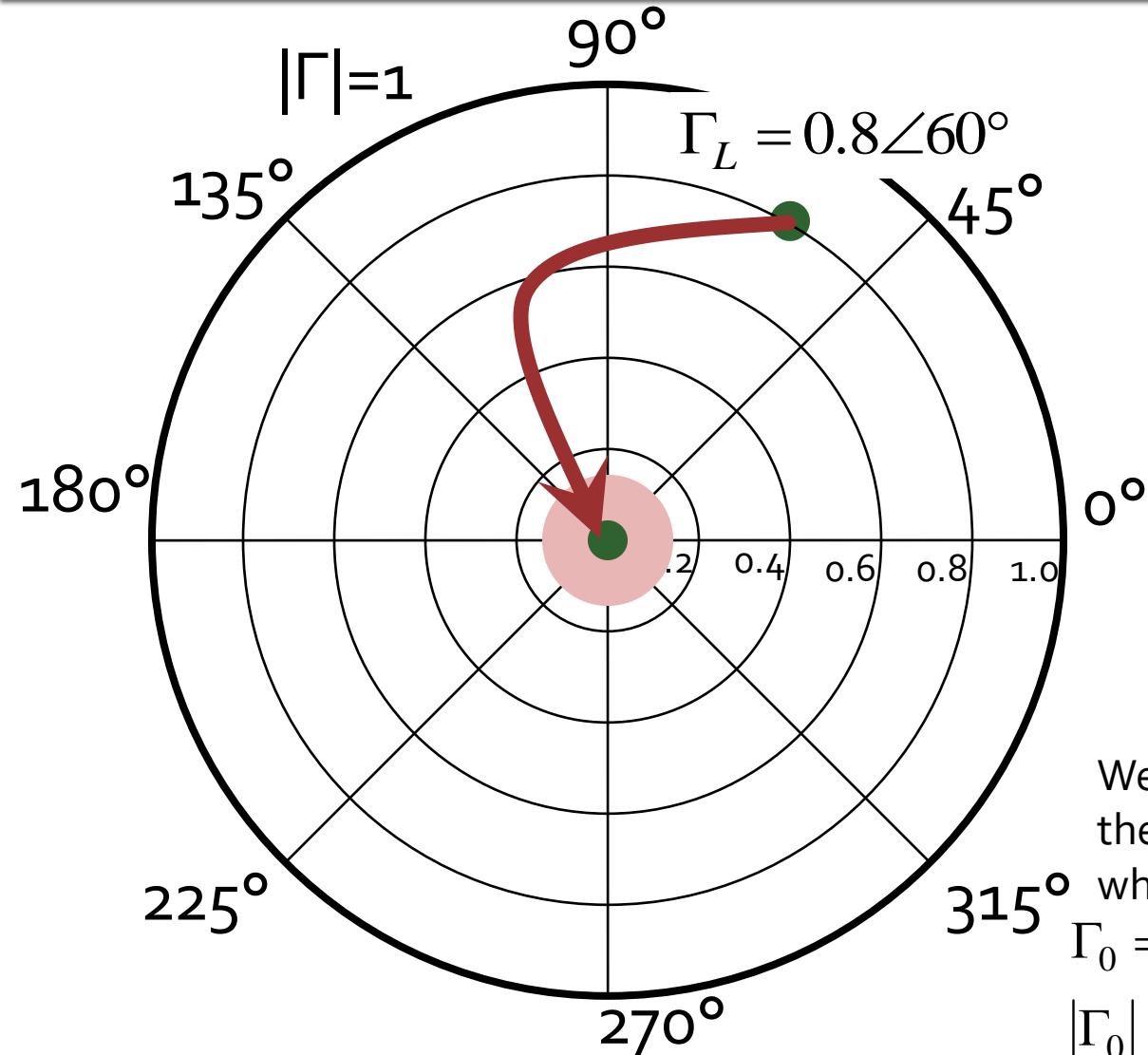
Impedance matching

Impedance Matching with lumped elements (L Networks)

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

The Smith Chart, reflection coefficient, impedance matching



Matching Z_L load to Z_0 source.
We normalize Z_L over Z_0

$$Z_L = 21.429\Omega + j \cdot 82.479\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

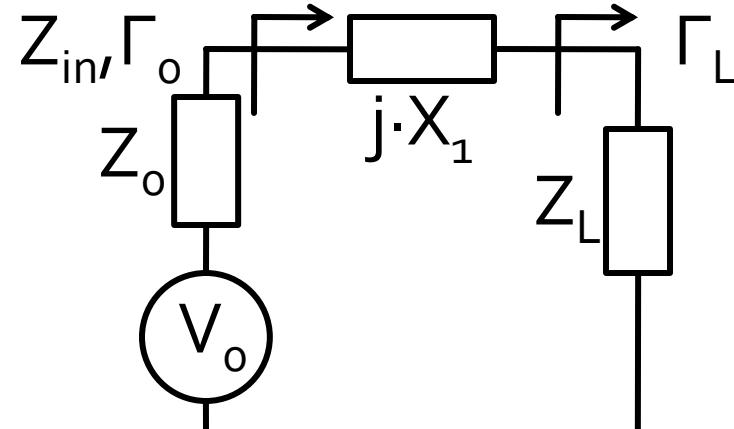
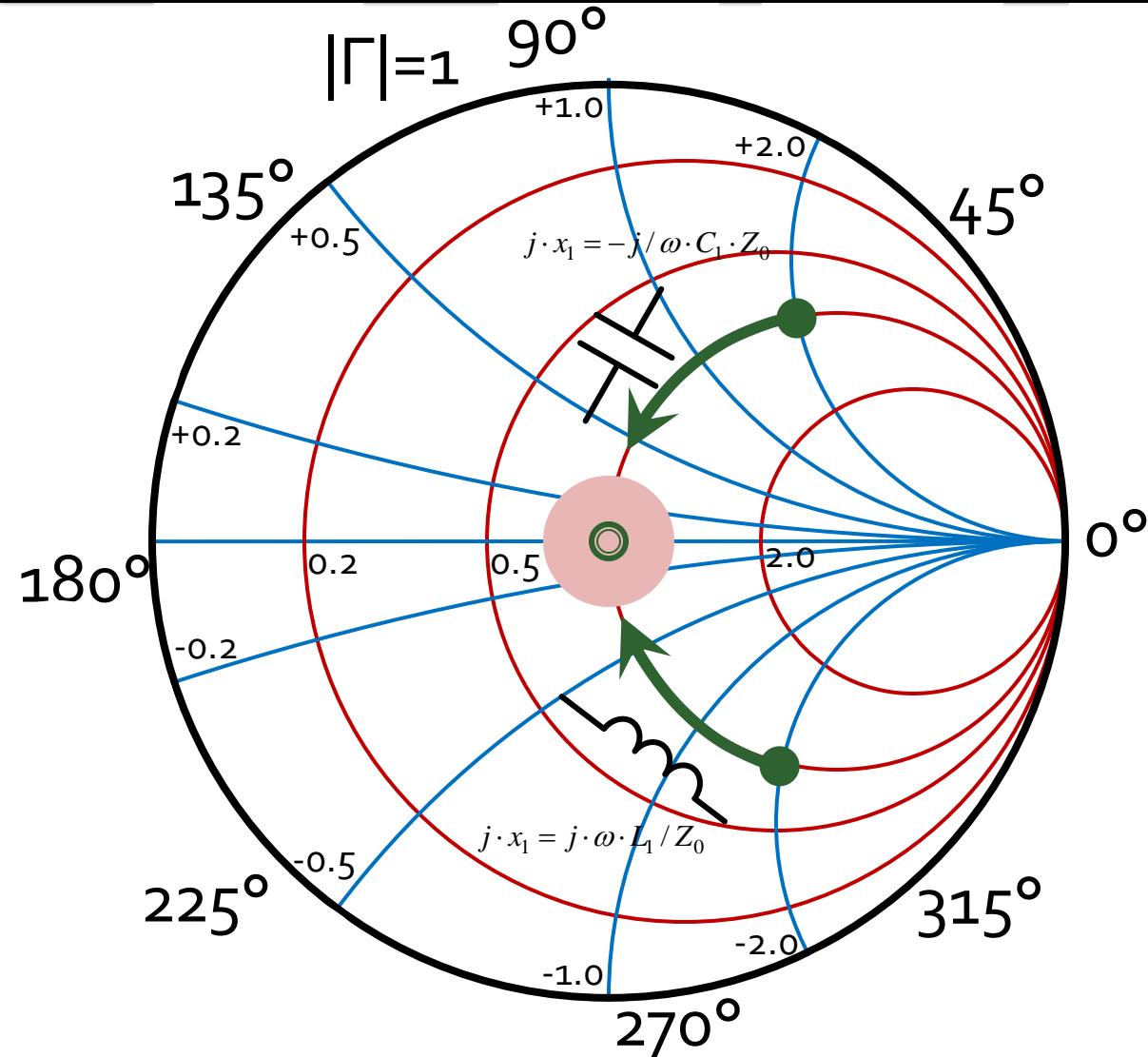
$$\Gamma_L = 0.8∠60°$$

We must move the point denoting
the reflection coefficient in the area
where with a Z_0 source we have:
 $\Gamma_0 = 0$ perfect match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

$|\Gamma_0| \leq \Gamma_m$ "good enough" match

Matching, series reactance



$$z_L = r_L + j \cdot x_L$$

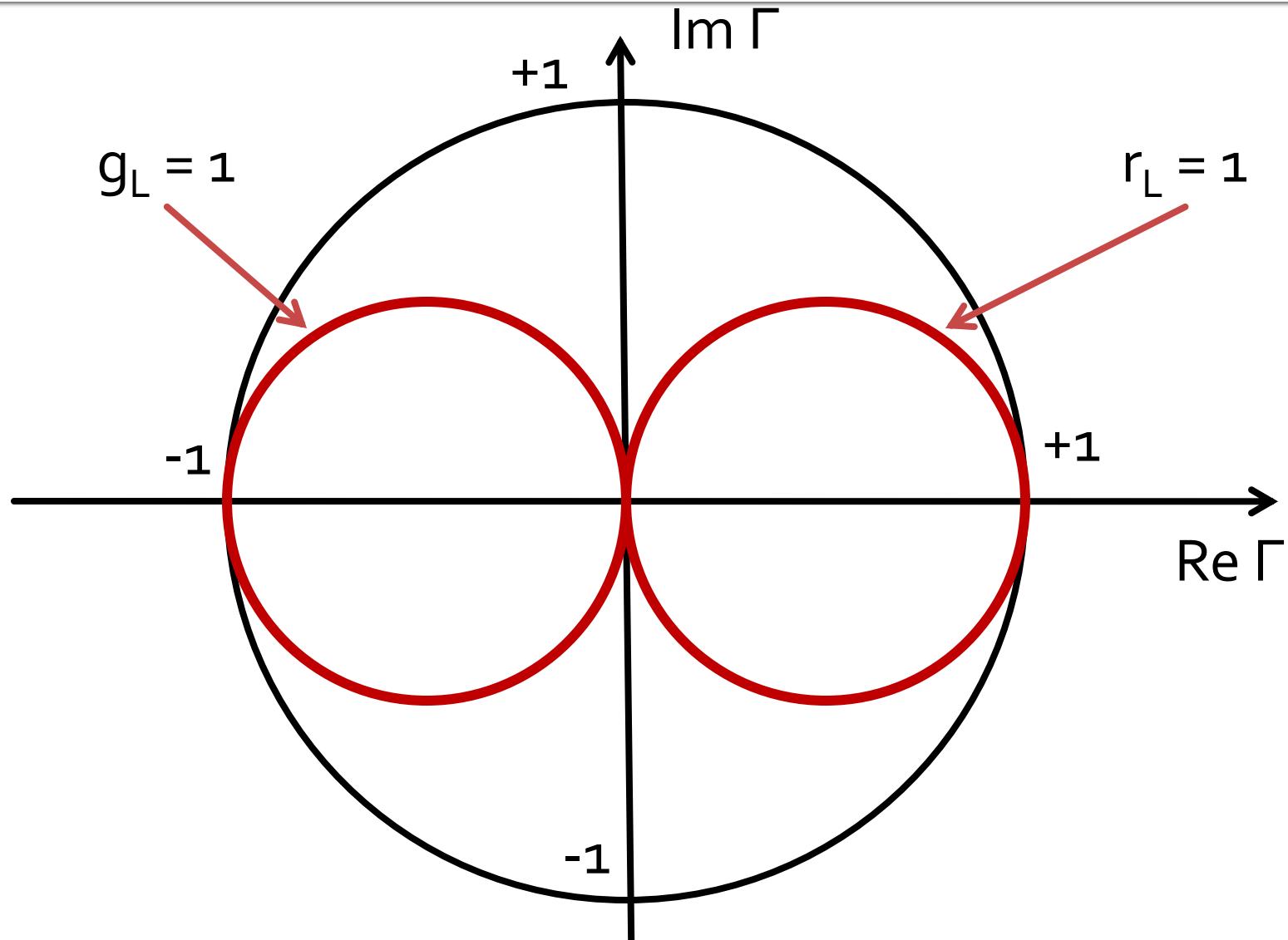
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

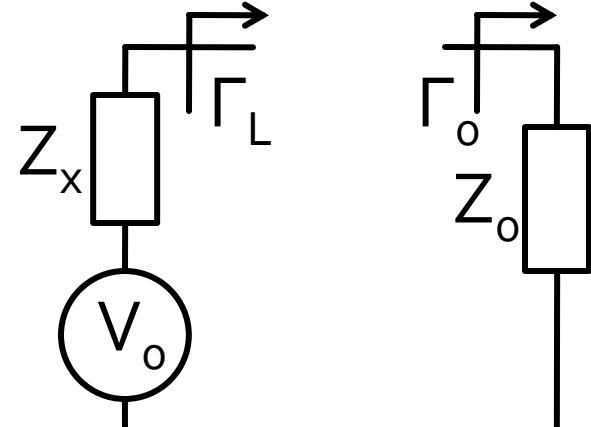
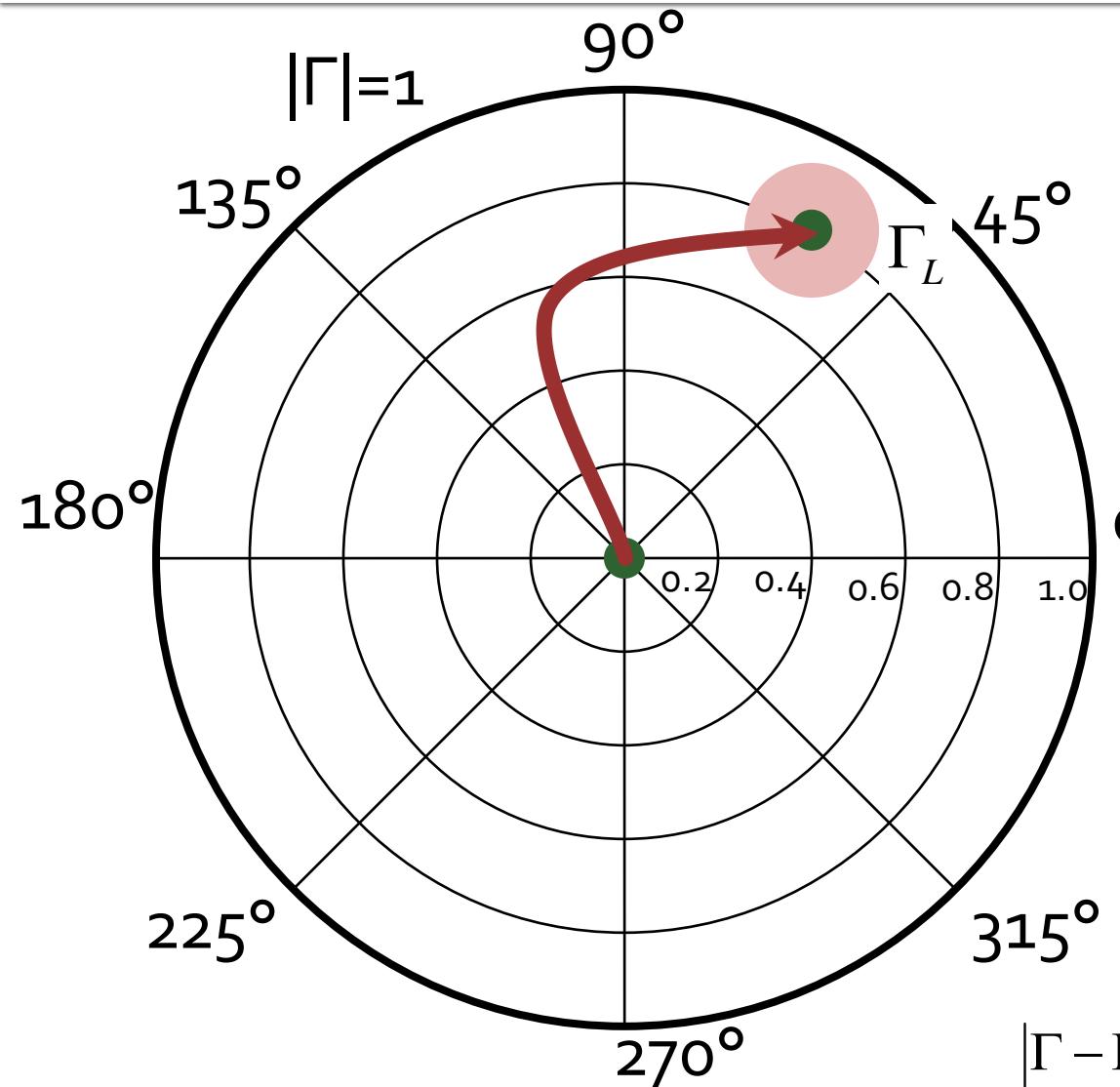
- Match can be obtained if and only if $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Smith chart, $r=1$ and $g=1$



The Smith Chart, matching, $Z_L = Z_o$



The source (eg. the transistor) having Z_x needs to see a certain reflection coefficient Γ_L towards the load Z_o

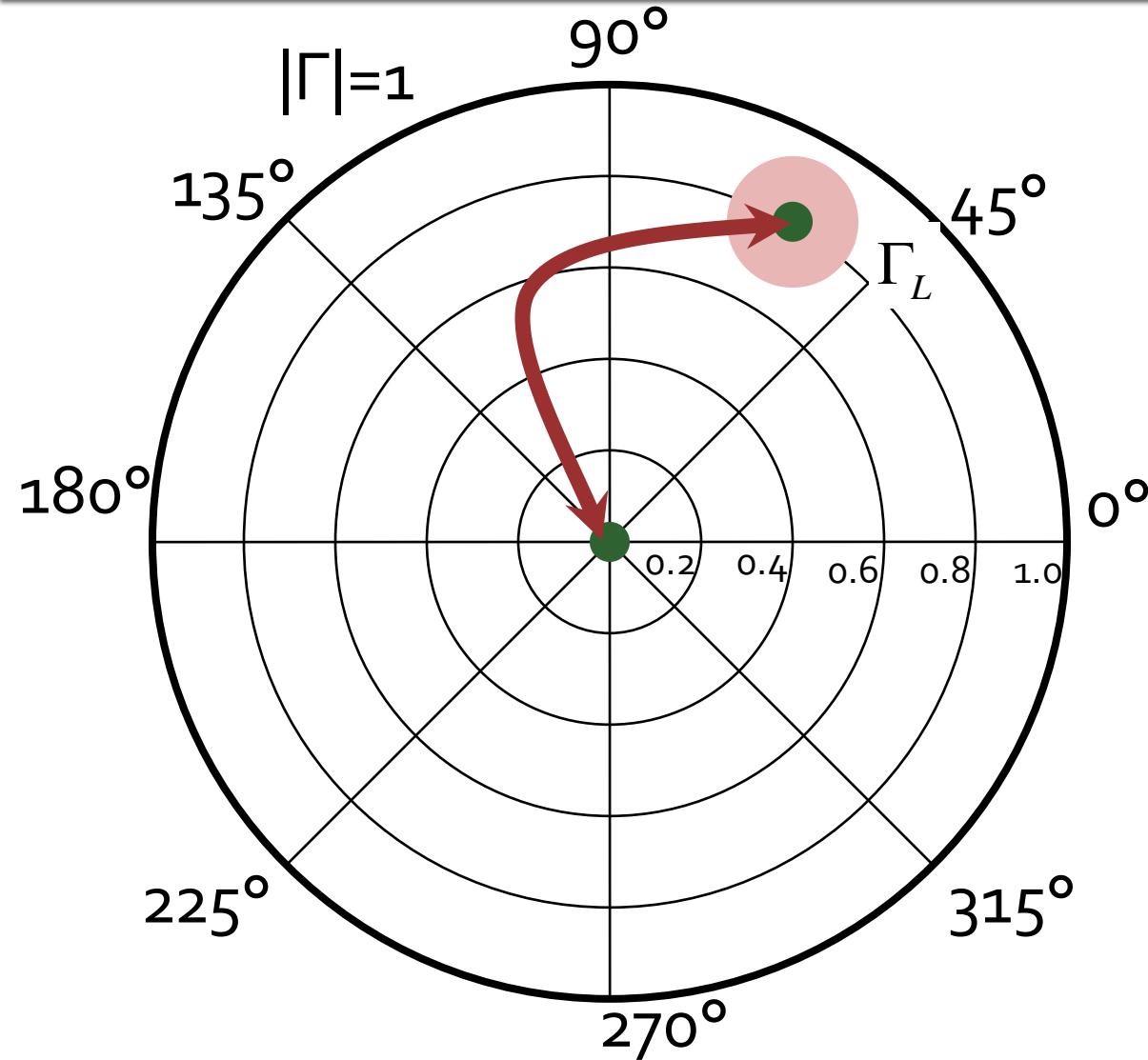
The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load ($\Gamma_o = 0$) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$|\Gamma - \Gamma_L| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching ,

$Z_L \neq Z_o$, $Z_L = Z_o$

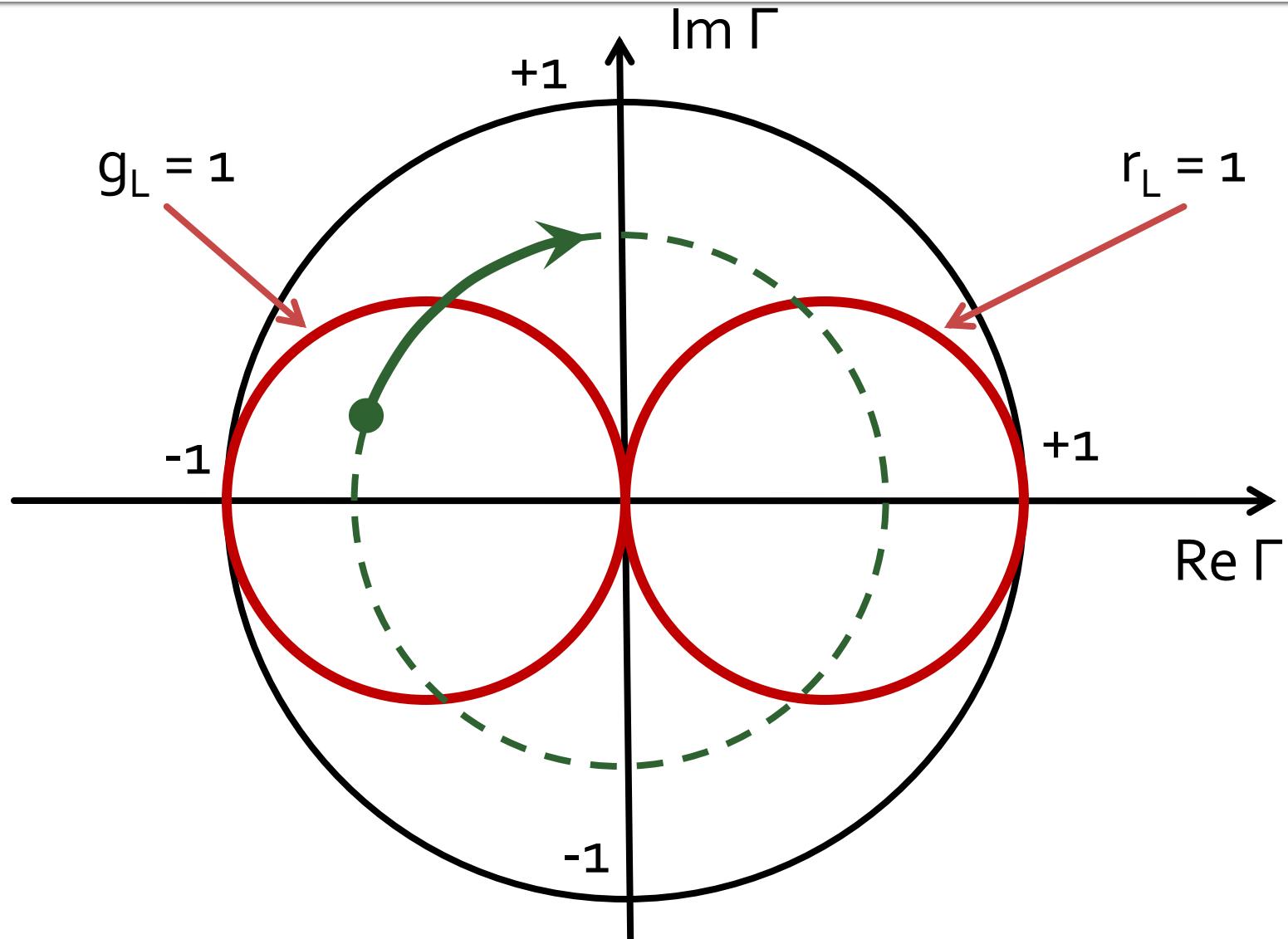


- The matching sections needed to move
 - Γ_L in Γ_o
 - Γ_o in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
 - **methods**
 - **formulae**

Impedance Matching

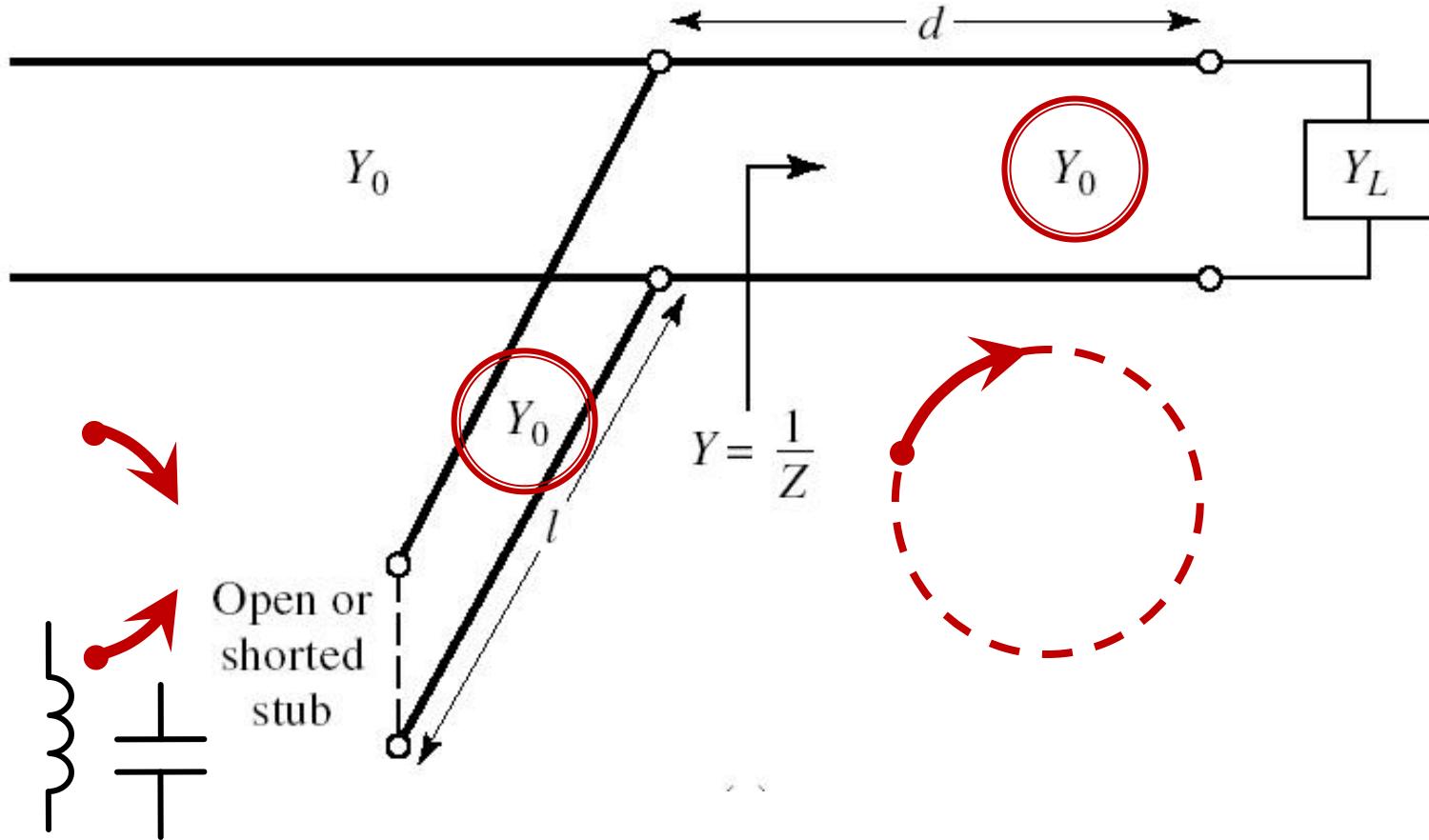
Impedance Matching with Stubs

Smith chart, $r=1$ and $g=1$



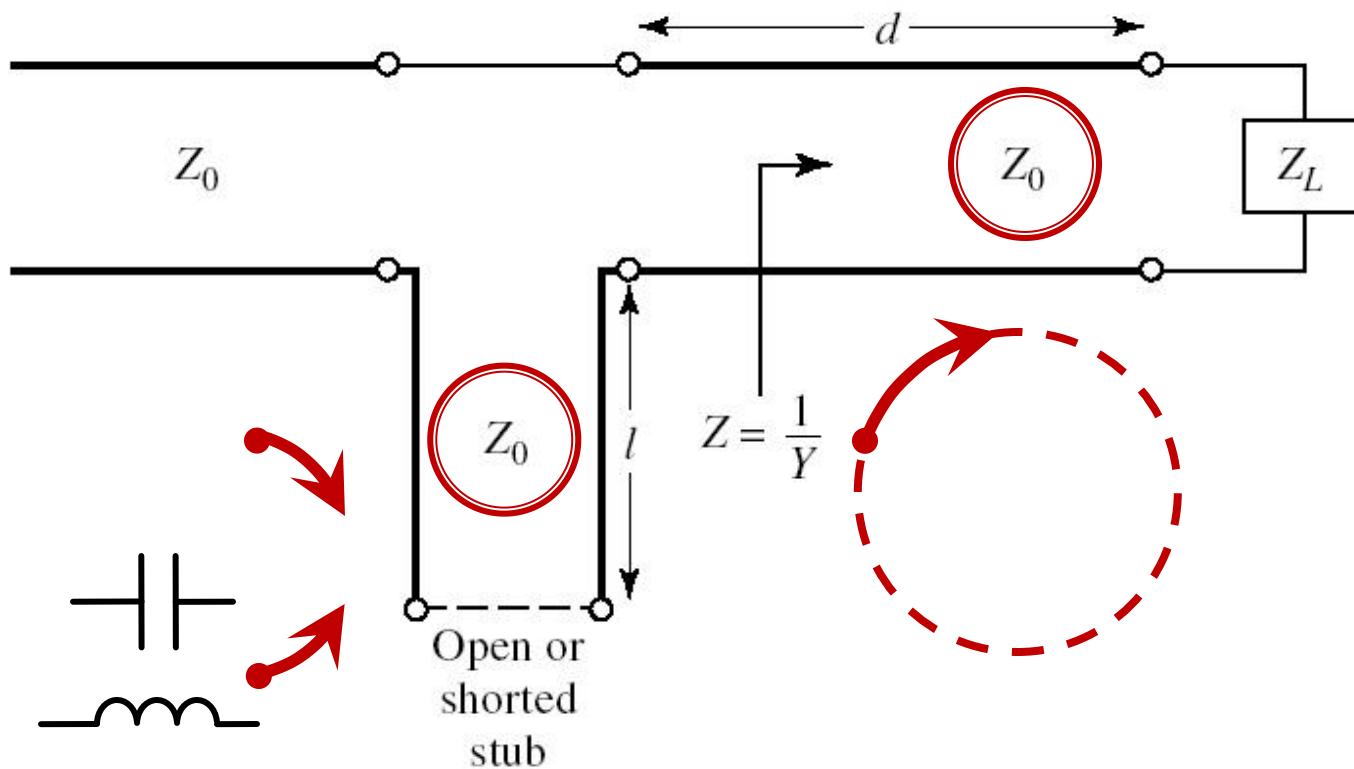
Single stub tuning

- Shunt Stub



Single stub tuning

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)

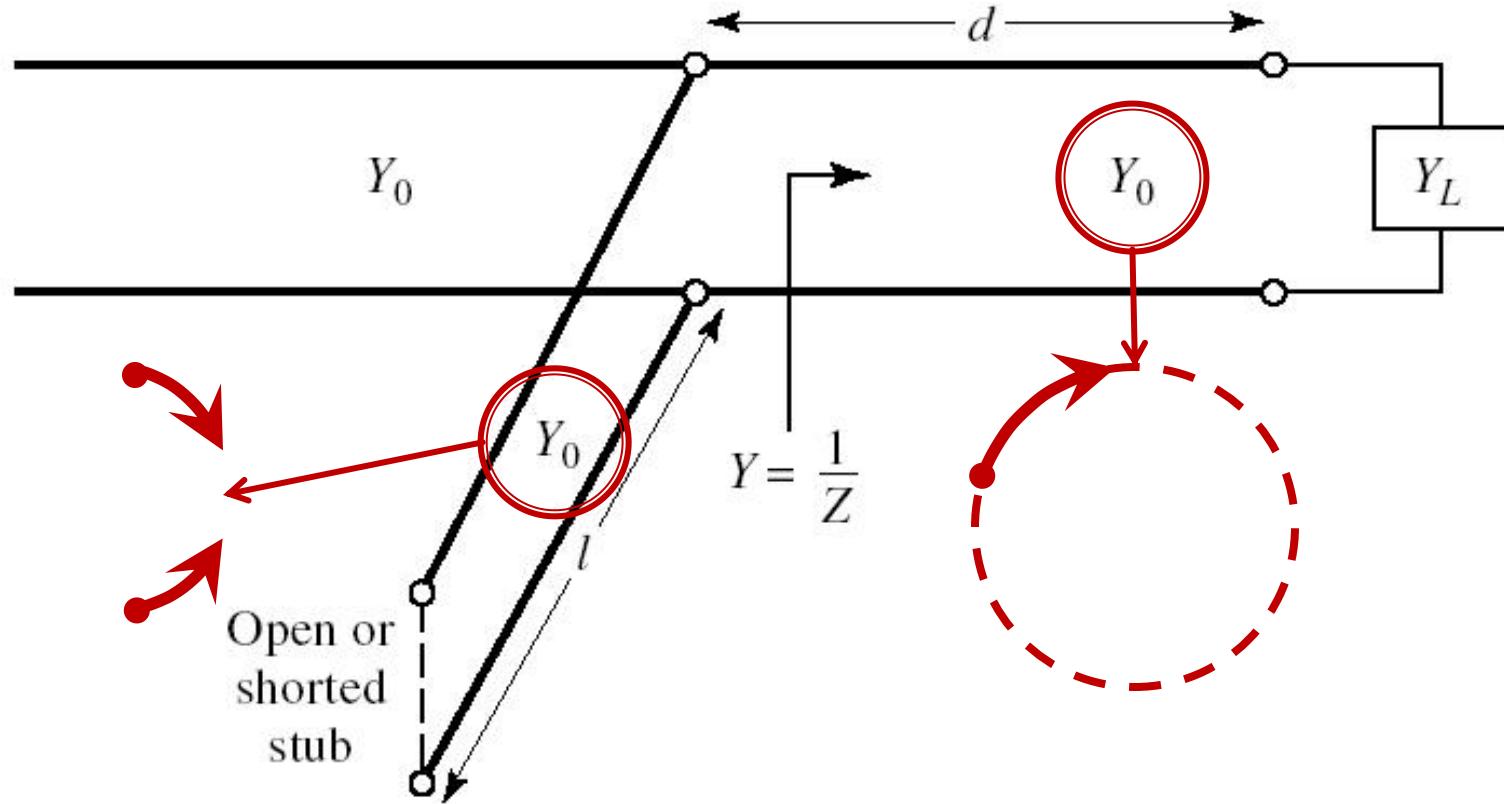


Analytical solutions

Exam / Project

Case 1, Shunt Stub

Shunt Stub



Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_s|$$

$$|\Gamma_s| = 0.593 \angle 46.85^\circ$$

$$|\Gamma_s| = 0.593; \quad \varphi = 46.85^\circ \quad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35^\circ$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

- “+” solution** ↓

$$(46.85^\circ + 2\theta) = +126.35^\circ \quad \theta = +39.7^\circ \quad \text{Im } y_s = \frac{-2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = -1.472$$
$$\theta_{sp} = \tan^{-1}(\text{Im } y_s) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

- “-” solution** ↓

$$(46.85^\circ + 2\theta) = -126.35^\circ \quad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\text{Im } y_s = \frac{+2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}} = +1.472 \quad \theta_{sp} = \tan^{-1}(\text{Im } y_s) = 55.8^\circ$$

Analytical solution, usage

$$(\varphi + 2\theta) = \begin{cases} +126.35^\circ \\ -126.35^\circ \end{cases} \quad \theta = \begin{cases} 39.7^\circ \\ 93.4^\circ \end{cases} \quad \text{Im}[y_s(\theta)] = \begin{cases} -1.472 \\ +1.472 \end{cases} \quad \theta_{sp} = \begin{cases} -55.8^\circ + 180^\circ = 124.2^\circ \\ +55.8^\circ \end{cases}$$

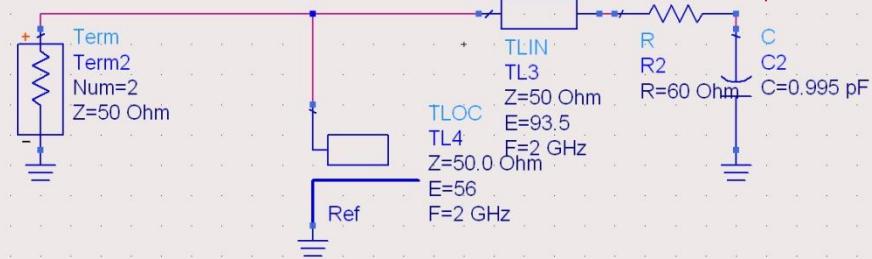
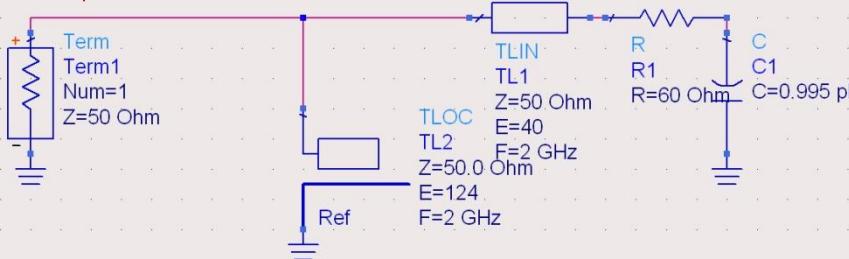
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

$$l_1 = \frac{39.7^\circ}{360^\circ} \cdot \lambda = 0.110 \cdot \lambda$$

$$l_2 = \frac{124.2^\circ}{360^\circ} \cdot \lambda = 0.345 \cdot \lambda$$

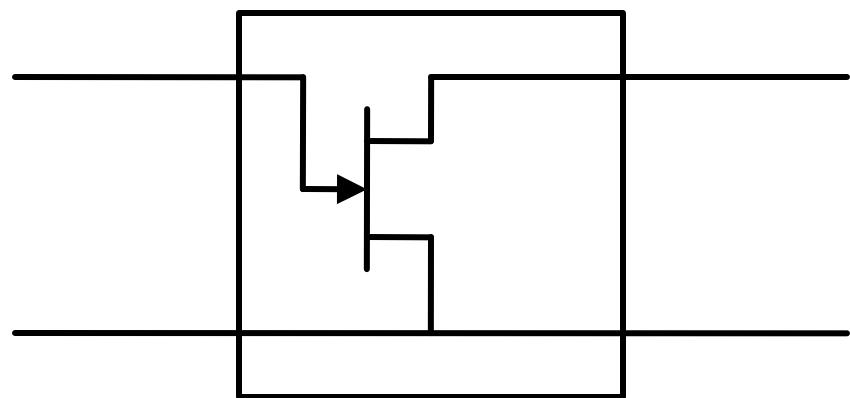
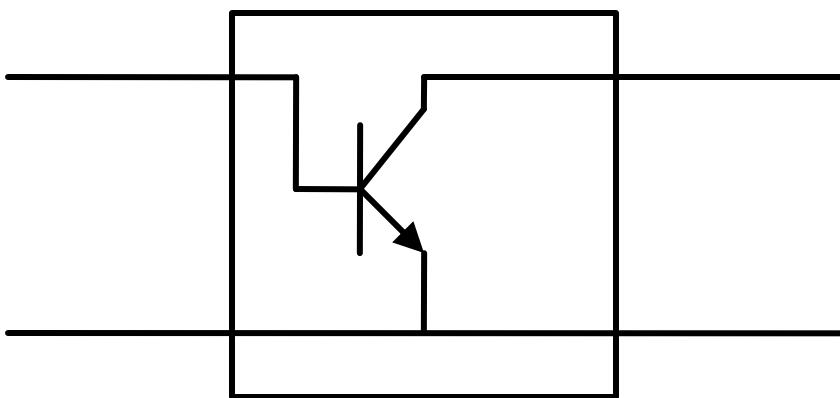
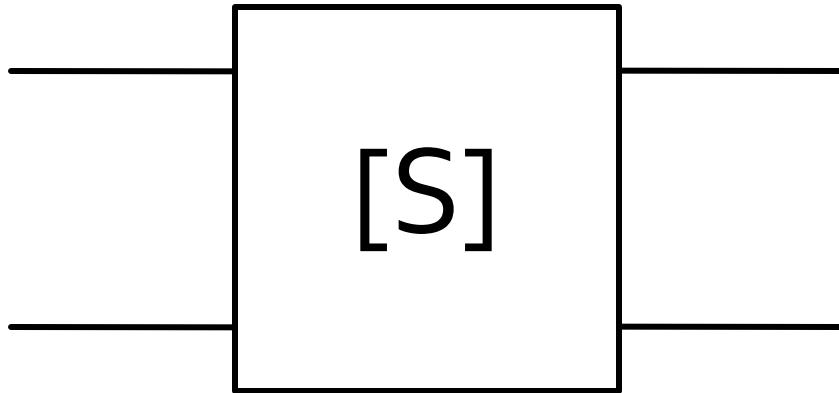
$$l_1 = \frac{93.4^\circ}{360^\circ} \cdot \lambda = 0.259 \cdot \lambda$$

$$l_2 = \frac{55.8^\circ}{360^\circ} \cdot \lambda = 0.155 \cdot \lambda$$

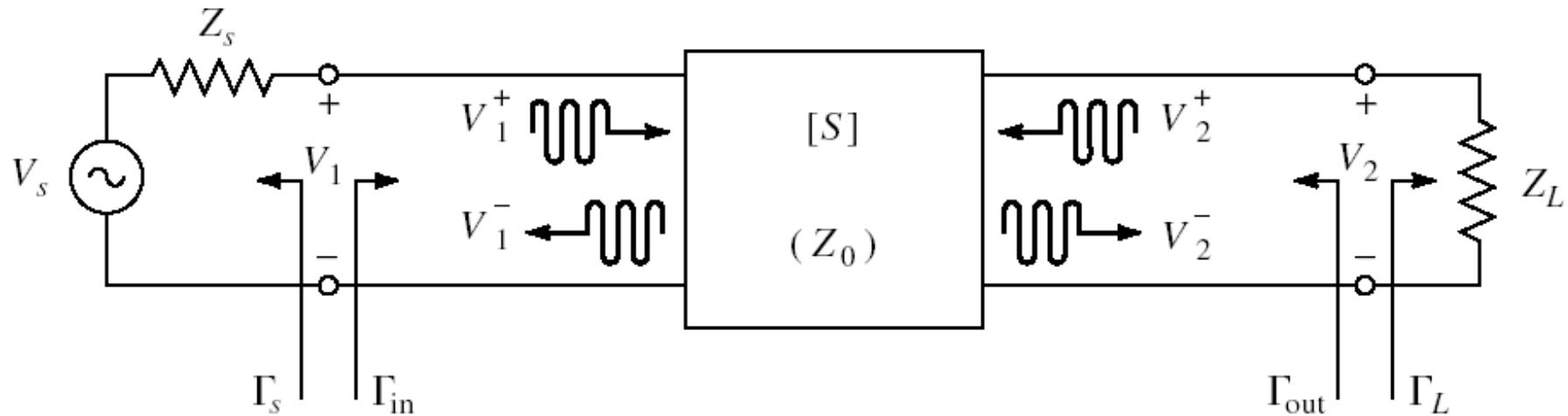


Microwave Amplifiers

S parameters for transistors



Amplifier as two-port



- Charaterized with S parameters
- normalized at Z_0 (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

Datasheets

NE46100

VCE = 5 V, Ic = 50 mA

FREQUENCY (MHz)	S ₁₁		S ₂₁		S ₁₂		S ₂₂		K	MAG ² (dB)
	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG		
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

VCE = 5 V, Ic = 100 mA

100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

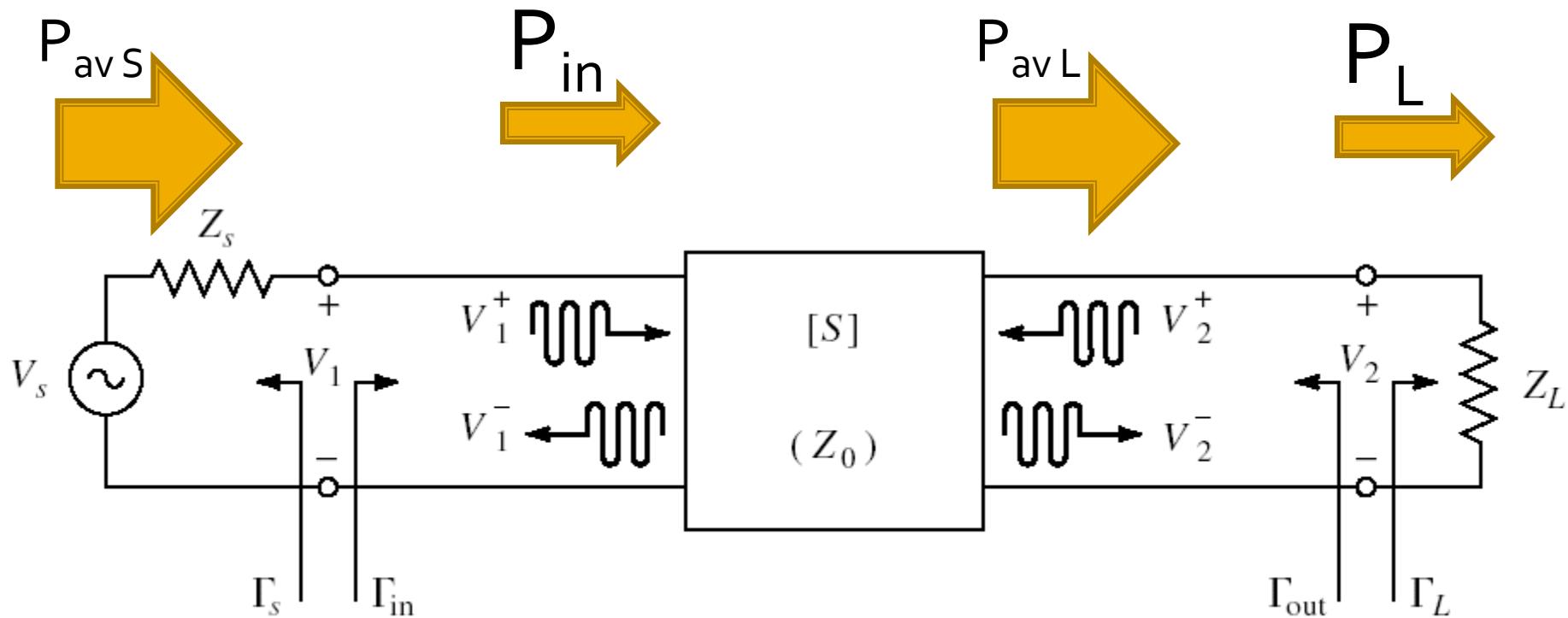
S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V  ID = 15 mA
# GHz S MA R 50
! f    S11      S21      S12      S22
! GHz  MAG  ANG  MAG  ANG  MAG  ANG  MAG  ANG
1.000 0.9800 -18.0 2.230 157.0 0.0240 74.0 0.6900 -15.0
2.000 0.9500 -39.0 2.220 136.0 0.0450 57.0 0.6600 -30.0
3.000 0.8900 -64.0 2.210 110.0 0.0680 40.0 0.6100 -45.0
4.000 0.8200 -89.0 2.230 86.0 0.0850 23.0 0.5600 -62.0
5.000 0.7400 -115.0 2.190 61.0 0.0990 7.0 0.4900 -80.0
6.000 0.6500 -142.0 2.110 36.0 0.1070 -10.0 0.4100 -98.0
!
! f    Fmin  Gammaopt rn/50
! GHz  dB  MAG  ANG  -
2.000  1.00 0.72 27 0.84
4.000  1.40 0.64 61 0.58
```

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\ L}}{P_{av\ S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2)}{|1 - S_{22} \cdot \Gamma_L|^2 \cdot (1 - |\Gamma_{out}|^2)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\ S}} = \frac{|S_{21}|^2 \cdot (1 - |\Gamma_S|^2) \cdot (1 - |\Gamma_L|^2)}{|1 - \Gamma_S \cdot \Gamma_{in}|^2 \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

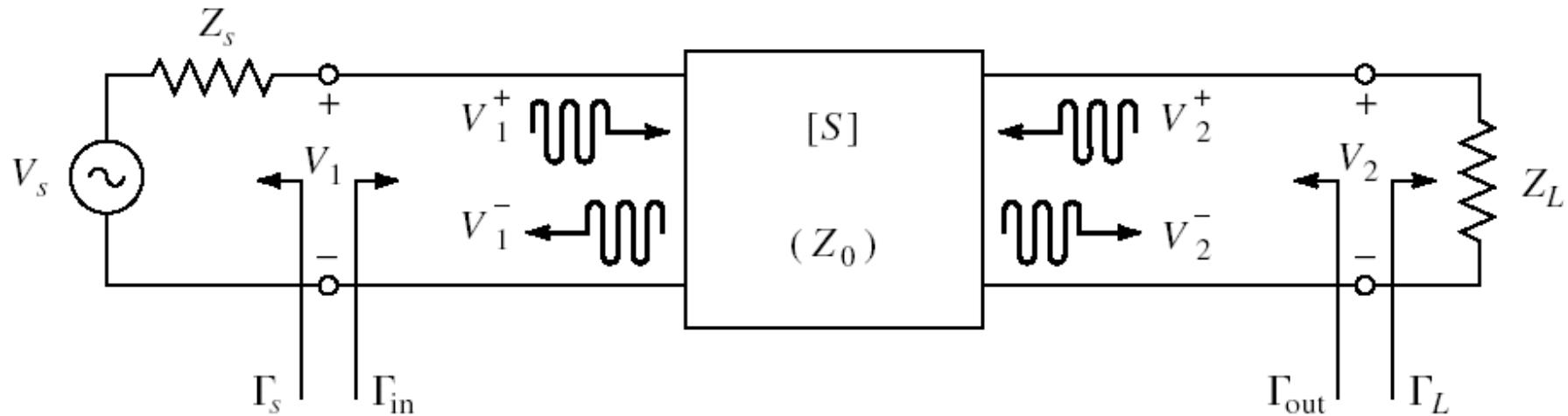
$$S_{12} \cong 0$$

$$\Gamma_{in} = S_{11}$$



Input and output can be treated independently

Amplifier as two-port

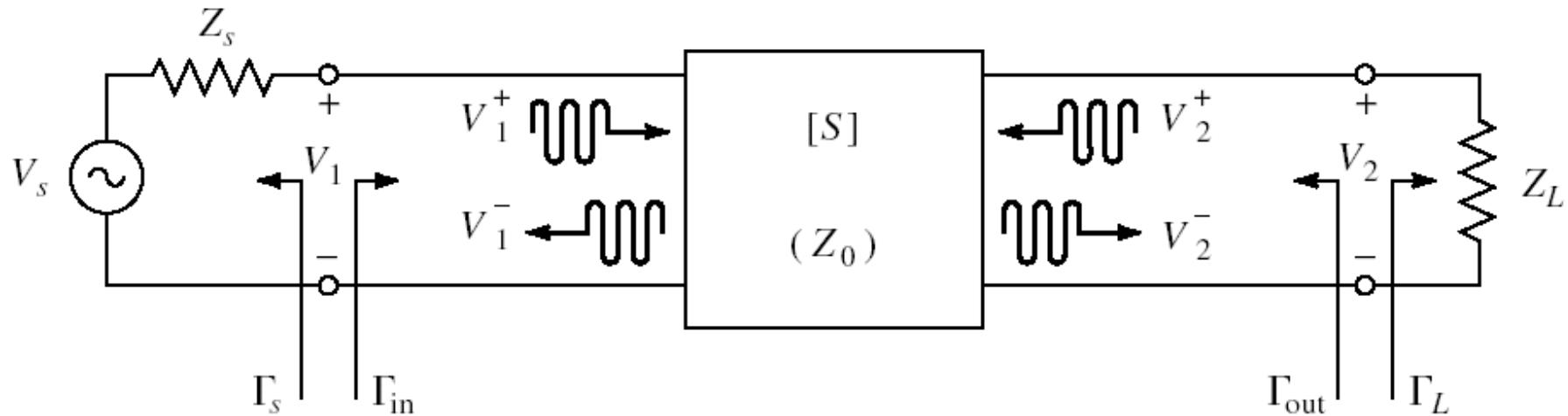


- For an amplifier two-port we are interested in:
 - stability
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Microwave Amplifiers

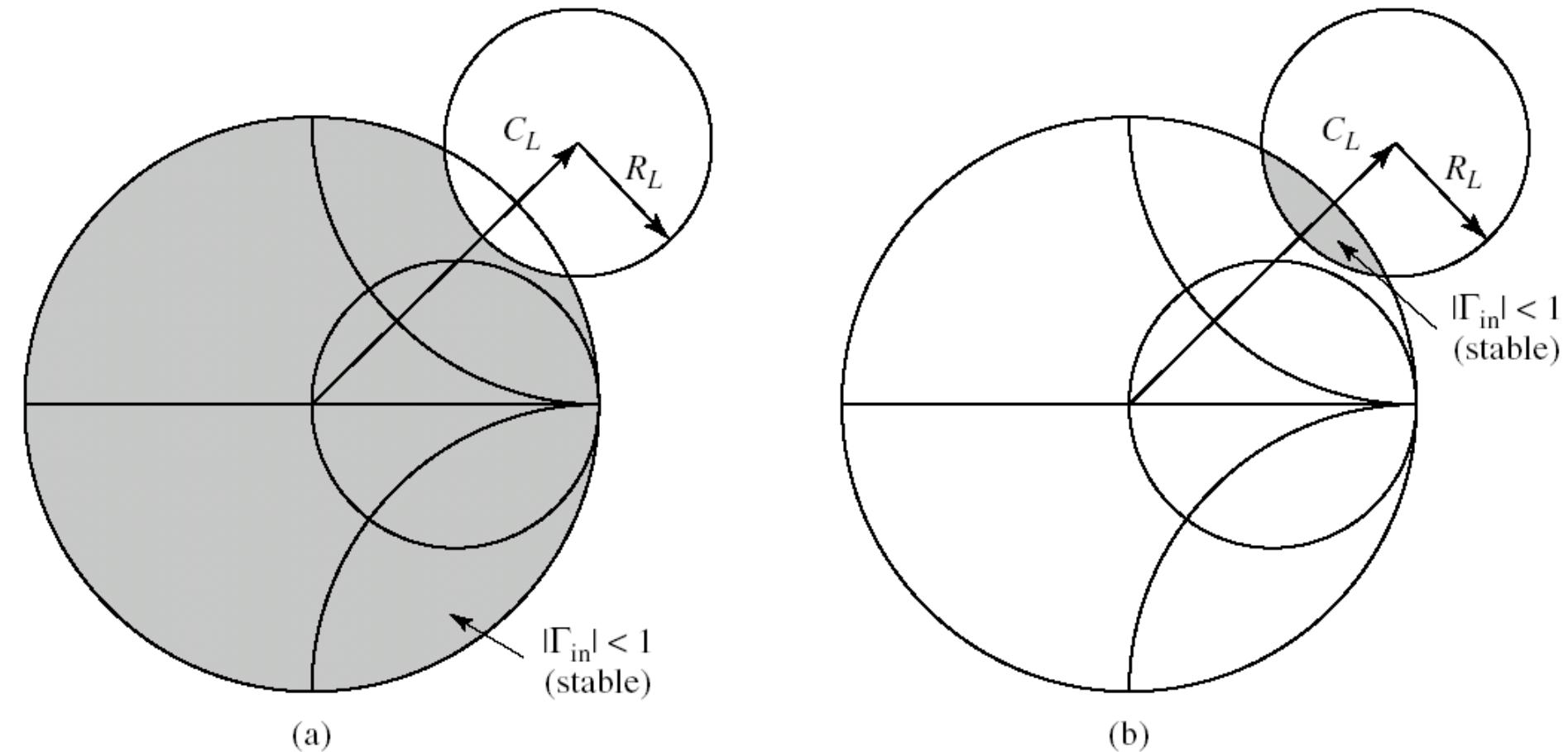
Stability

Amplifier as two-port



- For an amplifier two-port we are interested in:
 - **stability**
 - power gain
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

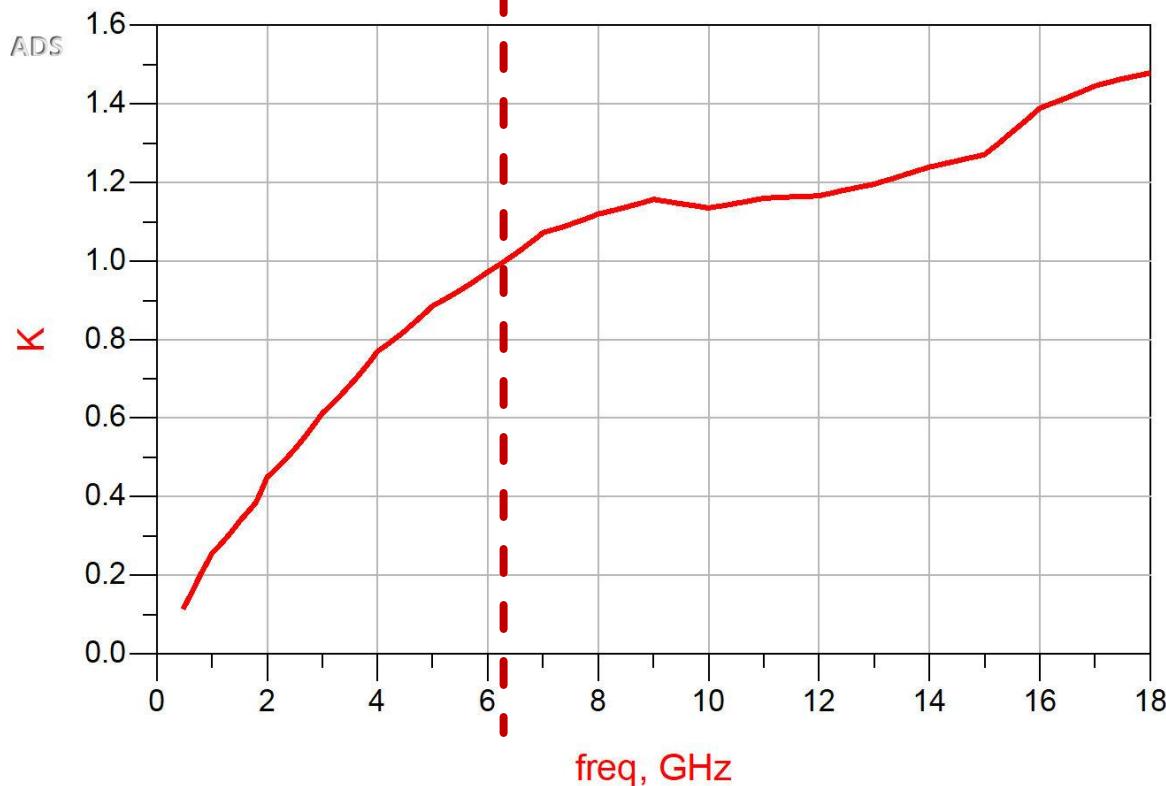
Output stability circle (CSOUT)



- Two cases possible: (a) stable outside/ (b) stable inside

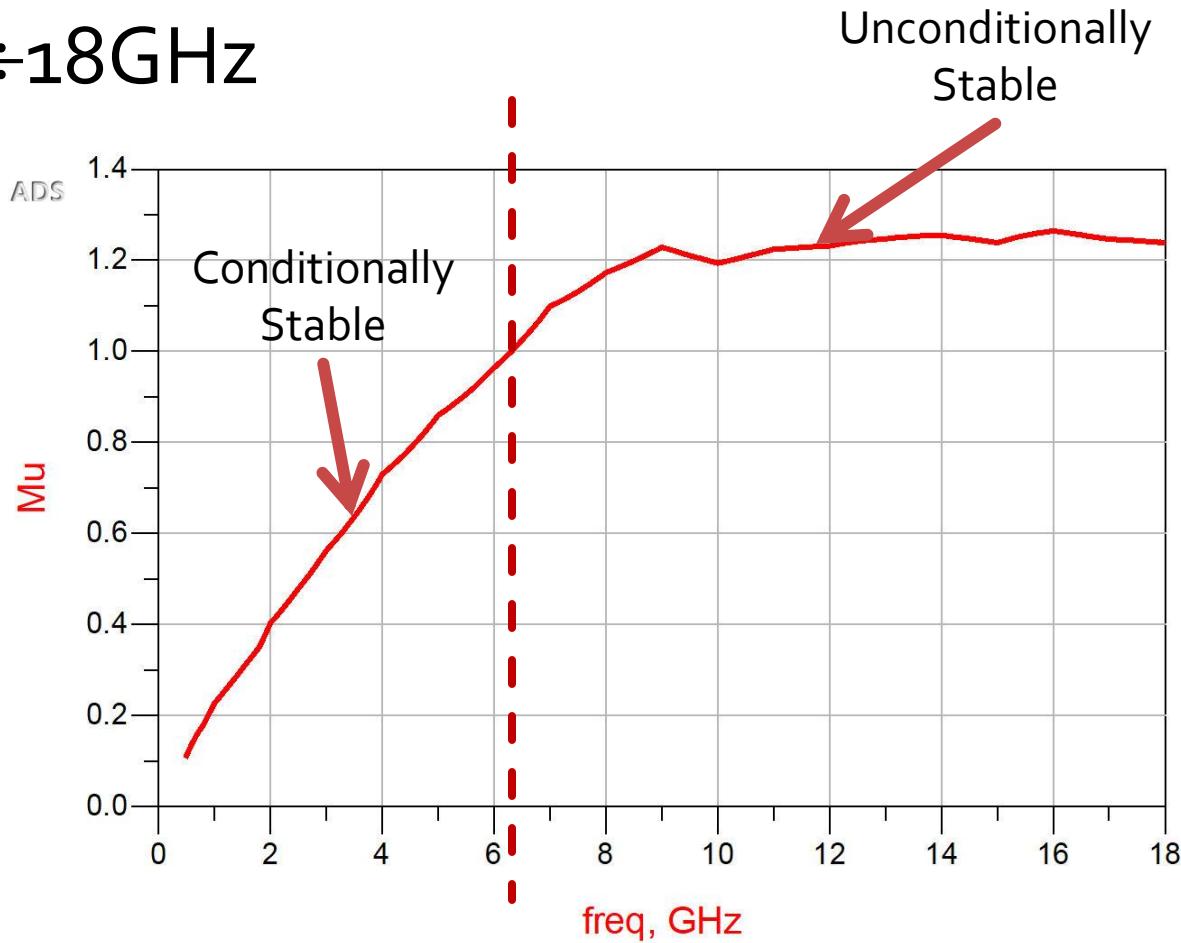
Rollet's condition

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @ $0.5 \div 18GHz$



μ Criterion

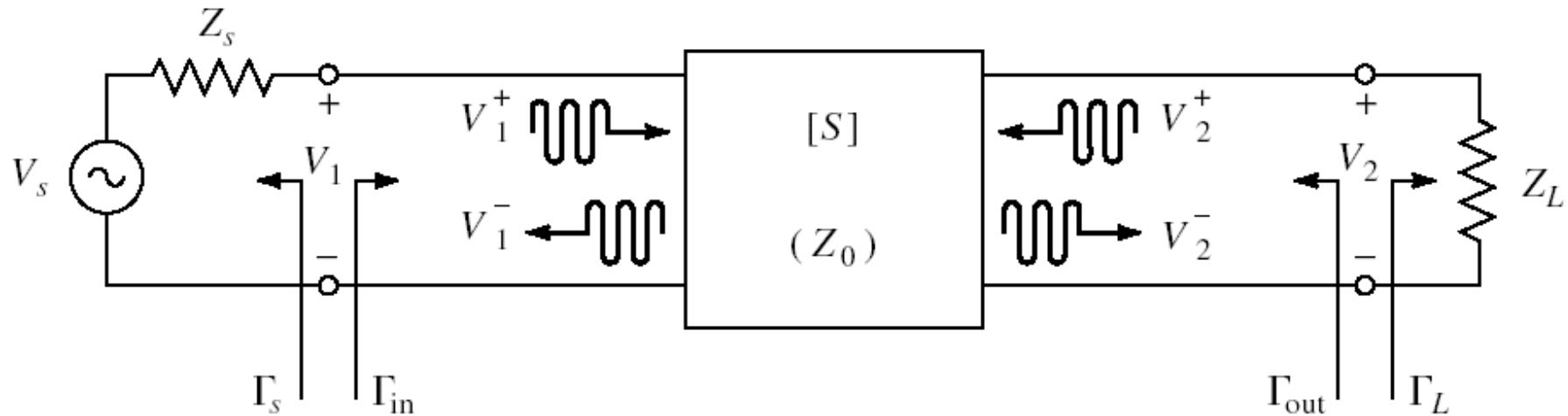
- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.
- @ $0.5 \div 18GHz$



Microwave Amplifiers

Power Gain of Microwave Amplifiers

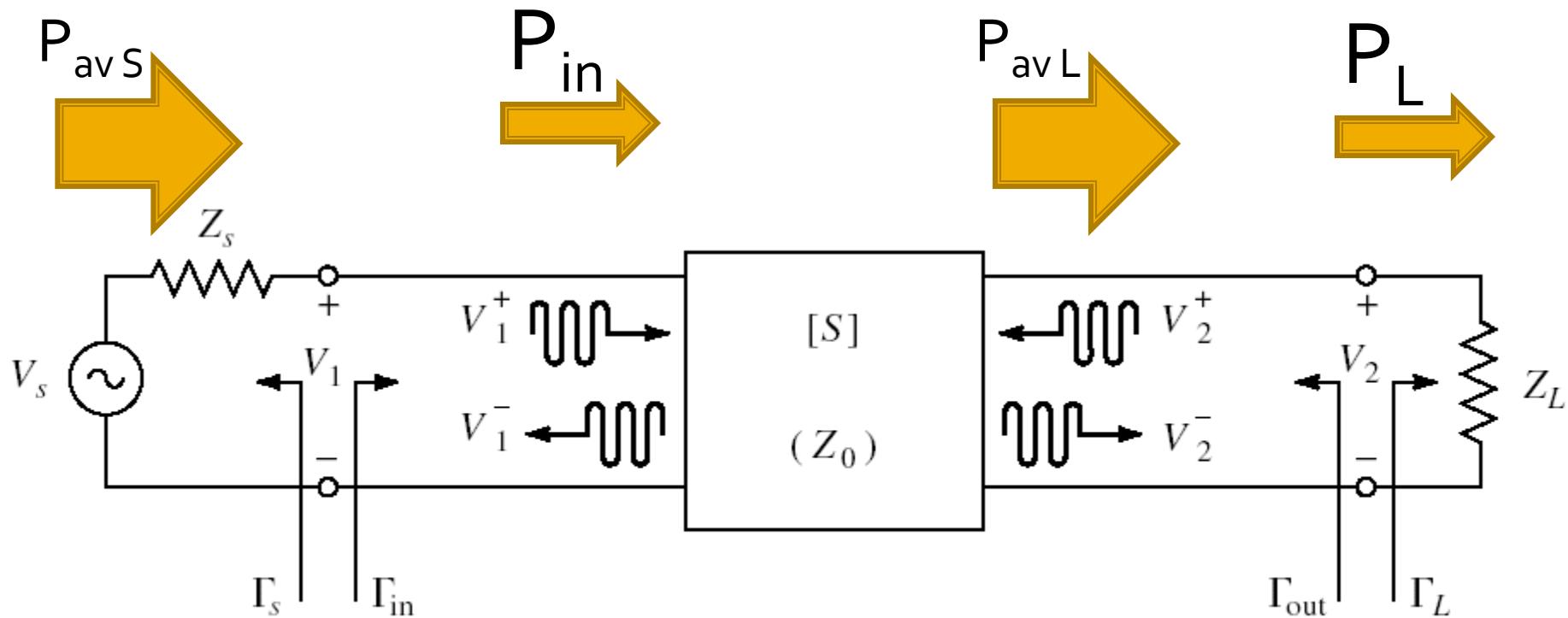
Amplifier as two-port



- For an amplifier two-port we are interested in:
 - stability
 - **power gain**
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Power / Matching

- Two ports in which matching influences the power transfer



Simultaneous matching

$$\rightarrow \Gamma_{in} = \Gamma_S^*$$
$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$
$$\Gamma_S^* = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$
$$\rightarrow \Gamma_{out} = \Gamma_L^*$$
$$\Gamma_{out} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$
$$\Gamma_L^* = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

■ We find Γ_S

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* \cdot S_{21}^*}{1/\Gamma_L^* - S_{22}^*}$$

$$\Gamma_L^* = \frac{S_{22} - \Delta \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_S \cdot (1 - |S_{22}|^2) + \Gamma_S^2 \cdot (\Delta \cdot S_{22}^* - S_{11}) = \Gamma_S \cdot (\Delta \cdot S_{11}^* \cdot S_{22}^* - |S_{22}|^2 - \Delta \cdot S_{12}^* \cdot S_{21}^*) + S_{11}^* \cdot (1 - |S_{22}|^2) + S_{12}^* \cdot S_{21}^* \cdot S_{22}$$

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and $|\Gamma| < 1$ solutions are those with “–” sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases}$$

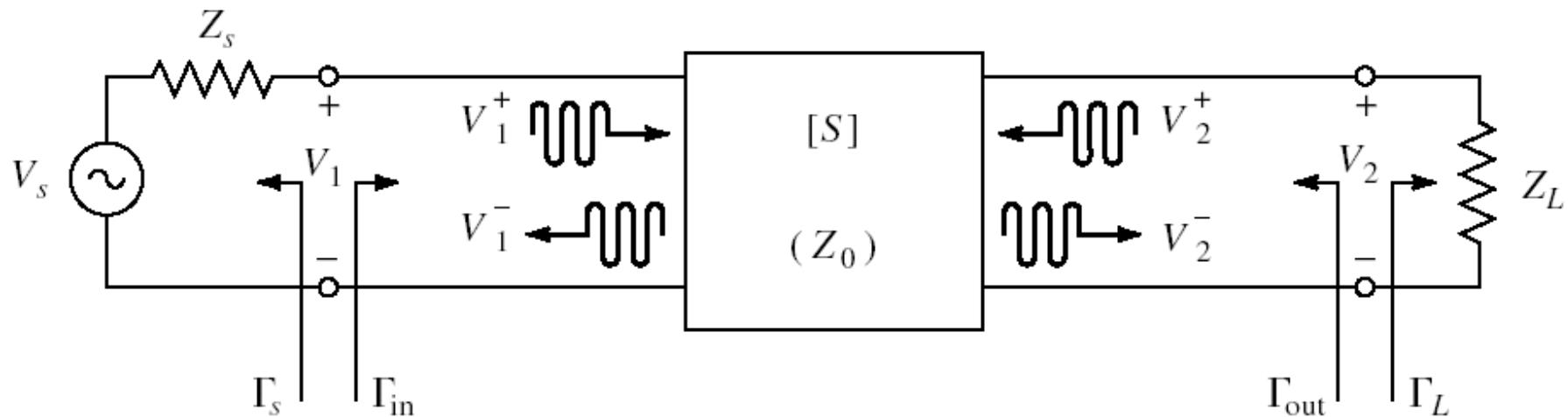
$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

Microwave Amplifiers

Design for Specified Gain

Amplifier as two-port



- For an amplifier two-port we are interested in:
 - stability
 - **power gain**
 - noise (sometimes – small signals)
 - linearity (sometimes – large signals)

Design for Specified Gain

- Assumes the amplifier device **unilateral**

$$G_{TU} = |S_{21}|^2 \cdot \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

Input and output can be treated independently

$$S_{12} \cong 0$$

$$\Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\ max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2}$$

Unilateral figure of merit

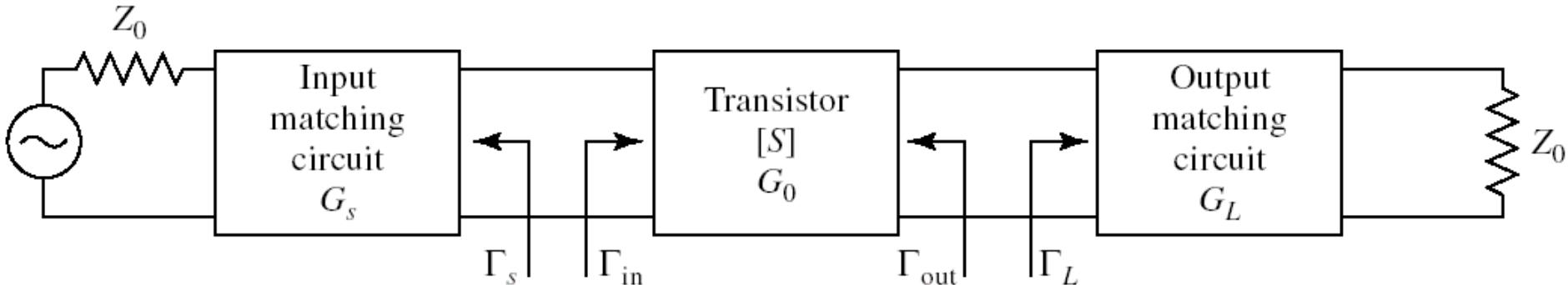
- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}$$
$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1 - |S_{11}|^2\right) \cdot \left(1 - |S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
 - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20 \cdot \log(1+U) < G_T [dB] - G_{TU} [dB] < -20 \cdot \log(1-U)$$

Design for Specified Gain



- In the unilateral assumption:

$$G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11} \cdot \Gamma_s|^2} \cdot |S_{21}|^2 \cdot \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$G_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11} \cdot \Gamma_s|^2}$$

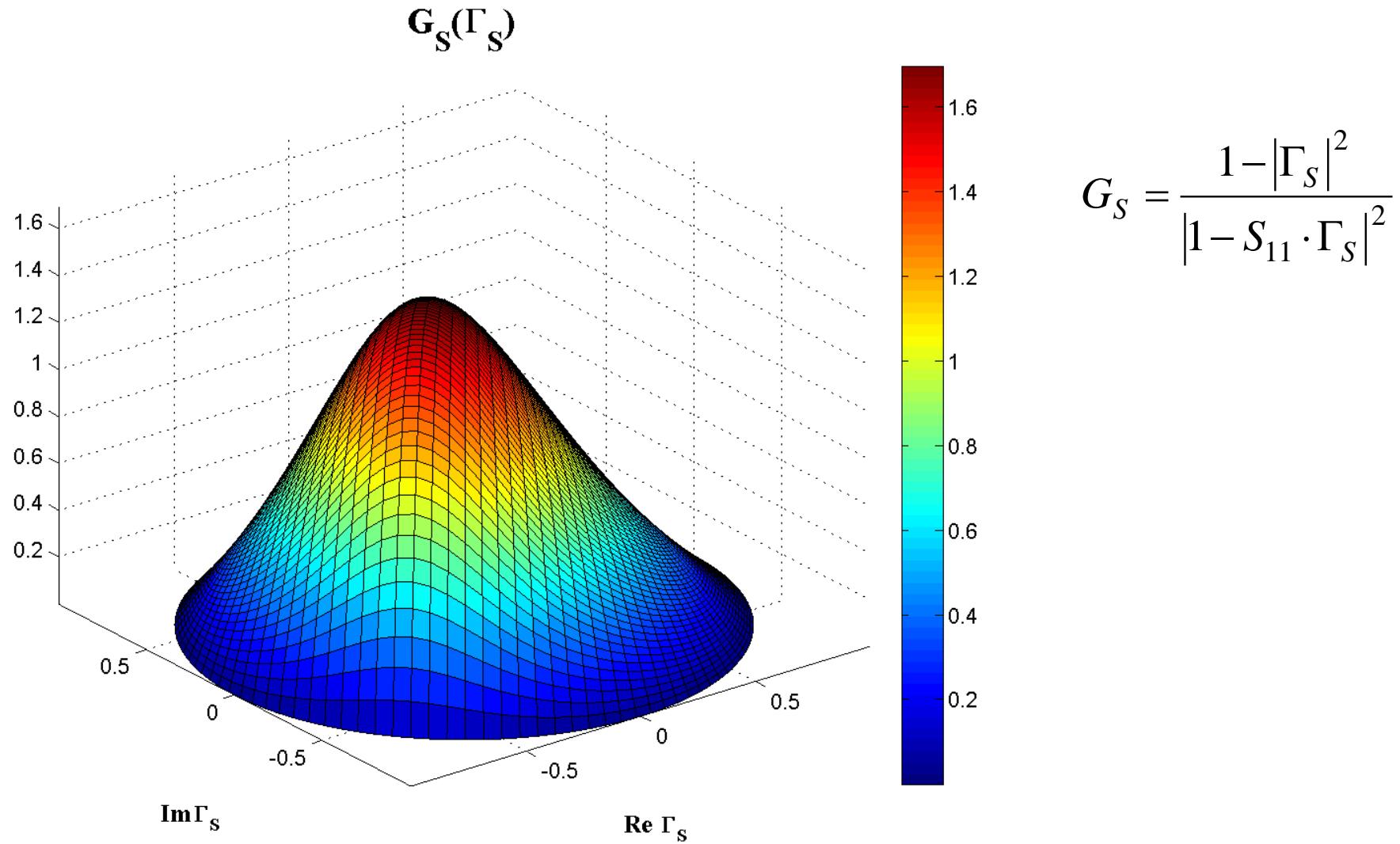
$$G_s = G_s(\Gamma_s)$$

$$G_0 = |S_{21}|^2$$

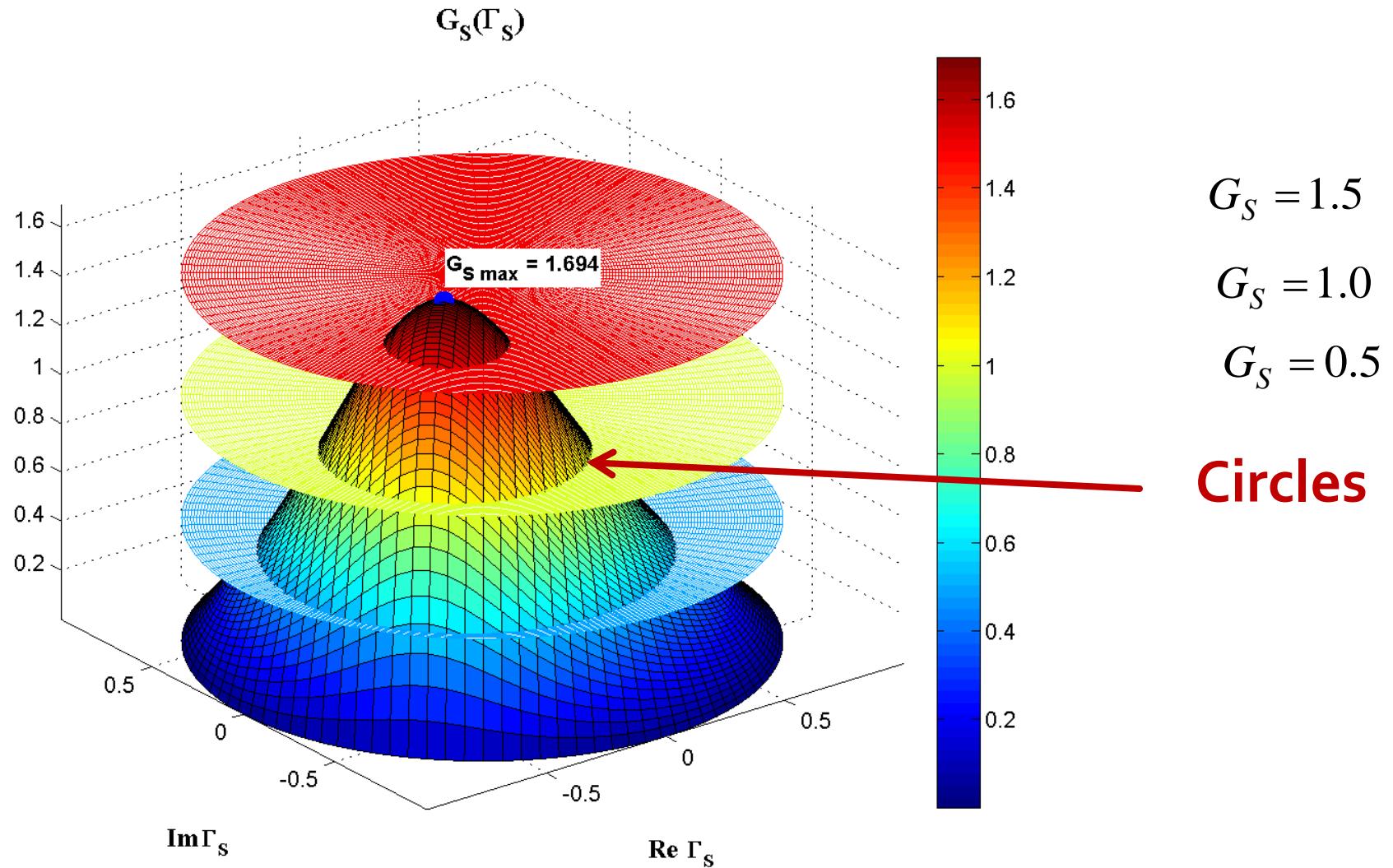
$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \cdot \Gamma_L|^2}$$

$$G_L = G_L(\Gamma_L)$$

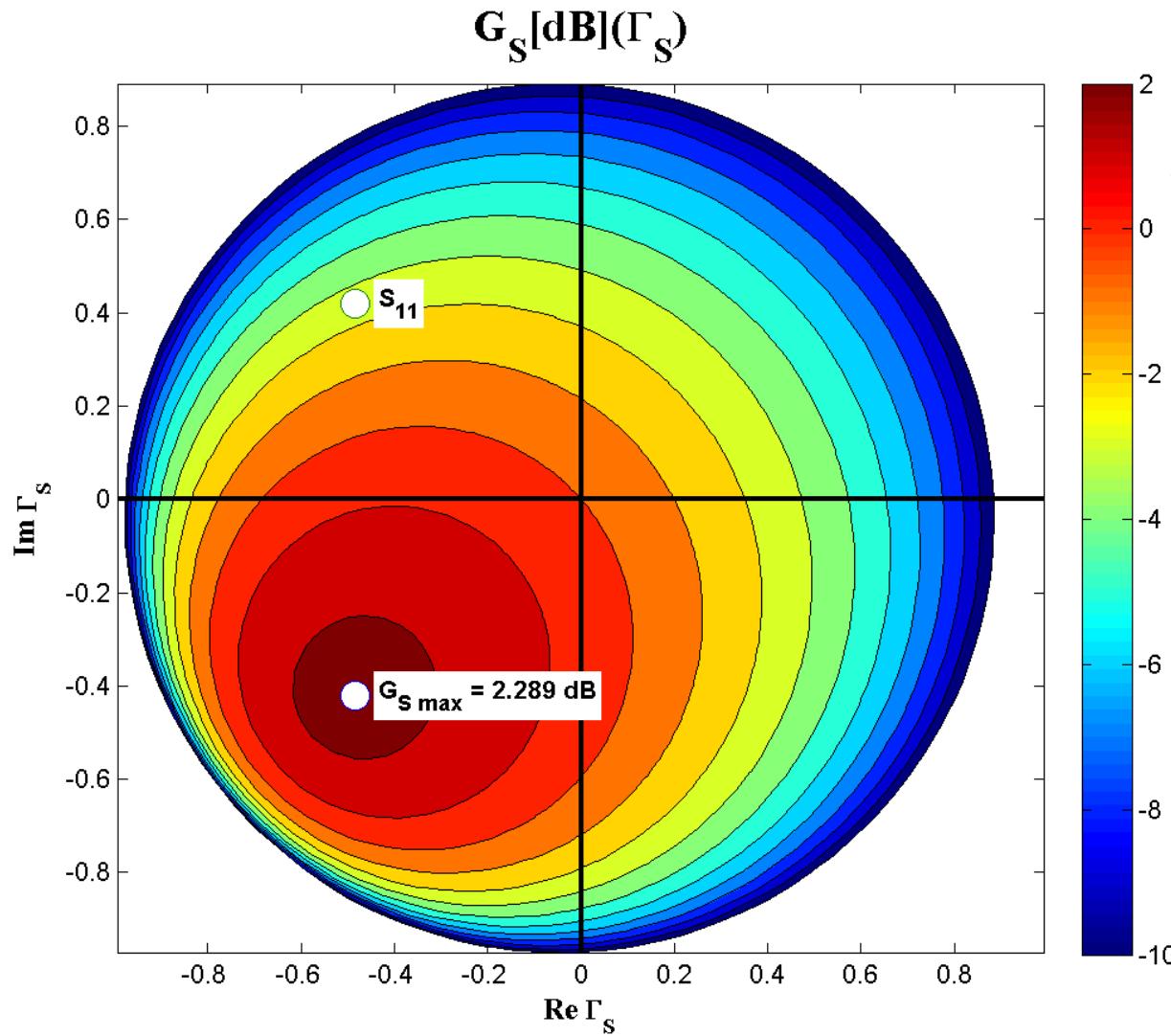
$\mathbf{G}_S(\Gamma_S)$



$G_S(\Gamma_S)$, constant value contours



$G_S[\text{dB}](\Gamma_S)$, constant value contours



$$G_S[\text{dB}] = 10 \cdot \log \left(\frac{1 - |\Gamma_S|^2}{|1 - S_{11} \cdot \Gamma_S|^2} \right)$$
$$G_S \text{ max} = G_S \Big|_{\Gamma_S = S_{11}^*}$$

Input section constant gain circles

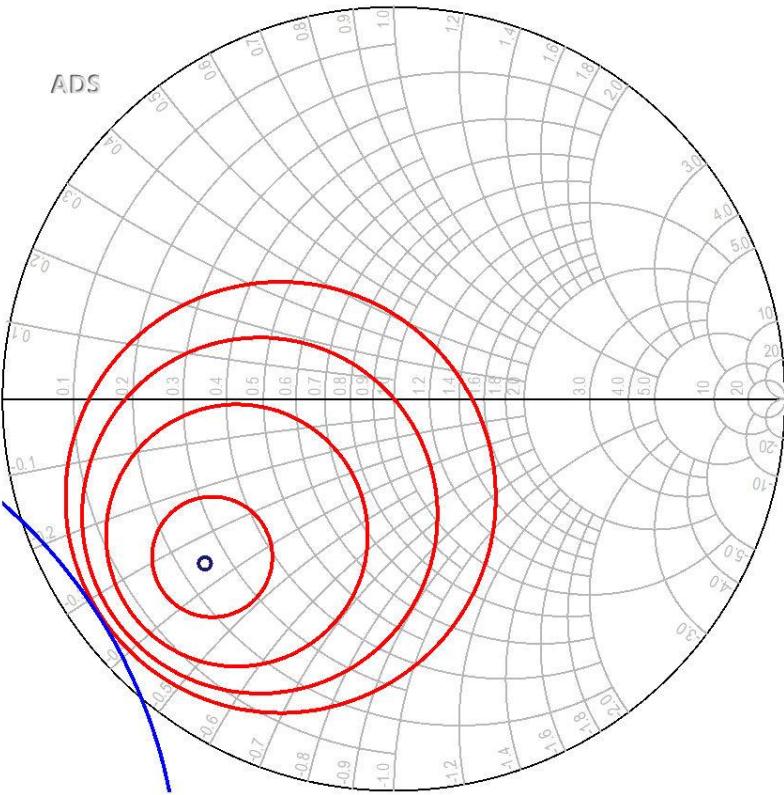
$$C_S = \frac{g_S \cdot S_{11}^*}{1 - (1 - g_S) \cdot |S_{11}|^2}$$

$$R_S = \frac{\sqrt{1 - g_S} \cdot (1 - |S_{11}|^2)}{1 - (1 - g_S) \cdot |S_{11}|^2}$$

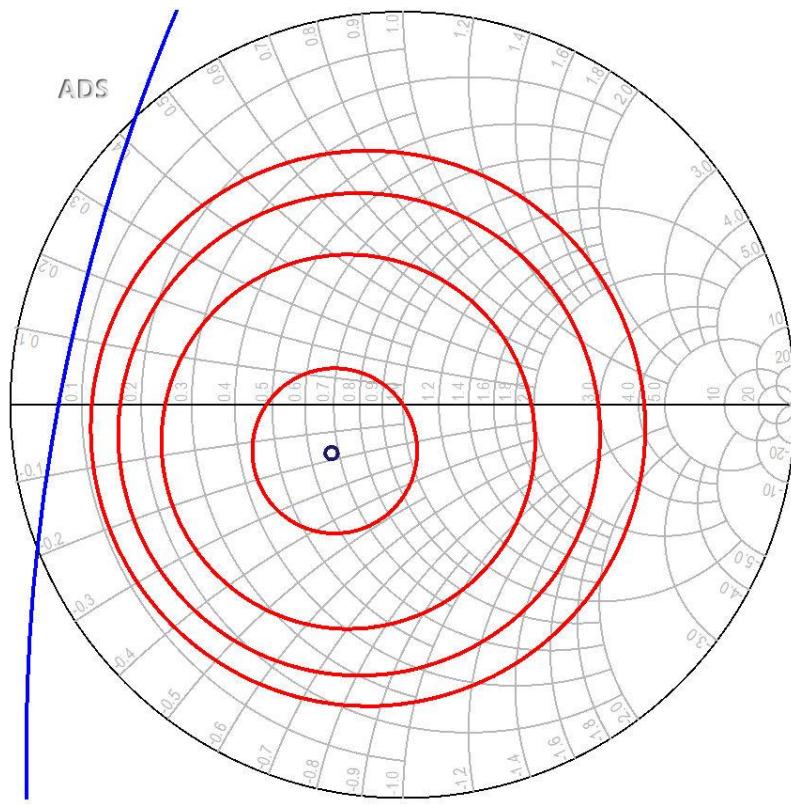
- The centers of each family of circles lie along straight lines given by the angle of $\Gamma_{S_{\max}} = S_{11}^*$
- Circles are plotted (traditionally, CAD) in **logarithmic scale** ([dB])
 - formulas are in **linear scale!**
- The circle for $G_S = 0 \text{ dB}$ will always pass through the origin of the complex plane (center of the Smith chart)

ADS

conj(S(1,1))
CSIN
CCCIN



conj(S(2,2))
CSOUT
CCCOUT

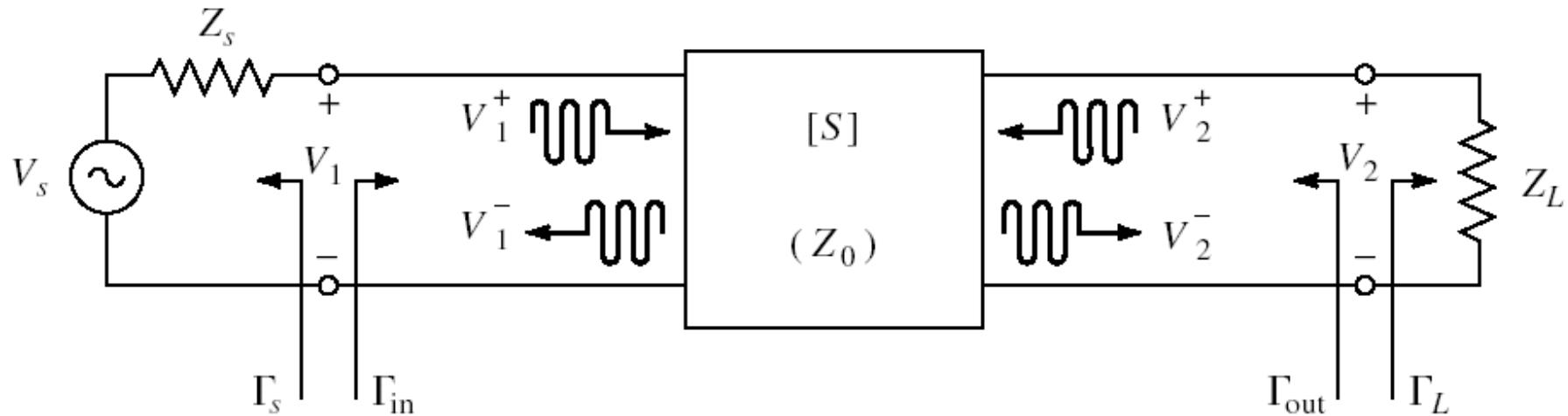


- Circles are plotted for requested values (**in dB!**)
- It is usefull to compute G_{Smax} and G_{Lmax} before
 - in order to request relevant circles

Microwave Amplifiers

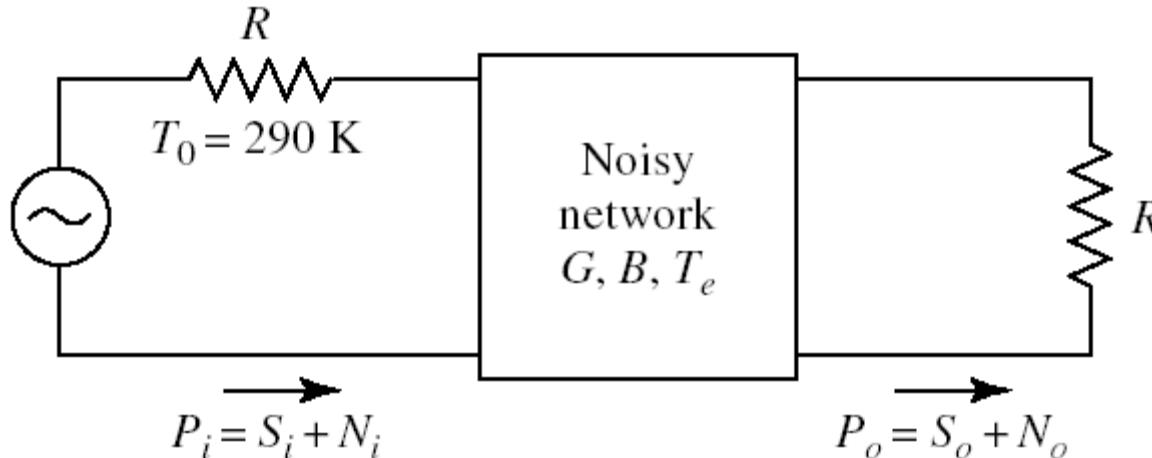
Low-Noise Amplifier Design

Amplifier as two-port



- For an amplifier two-port we are interested in:
 - stability
 - power gain
 - **noise** (sometimes – **small signals**)
 - linearity (sometimes – large signals)

Noise Figure F

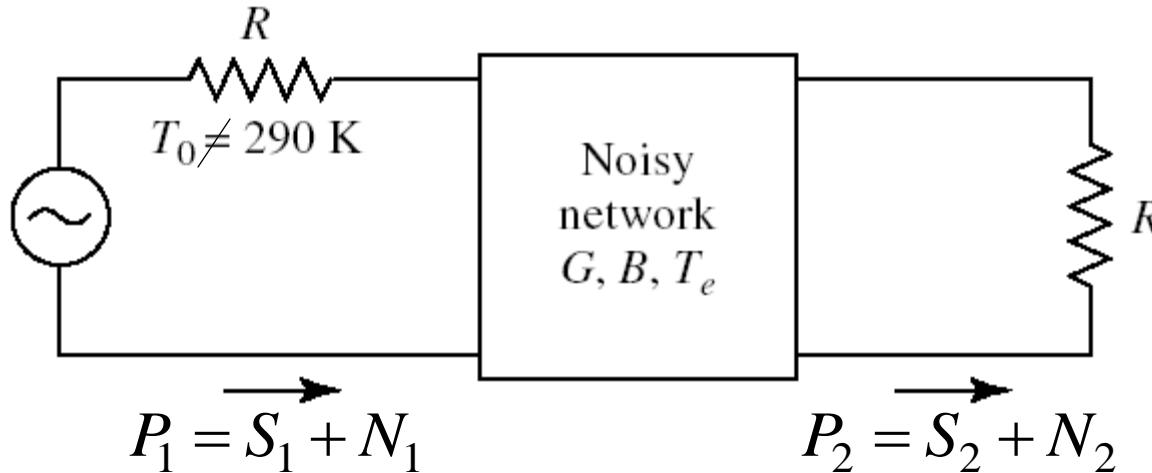


- The noise figure F , is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at $T_0 = 290 \text{ K}$ (reference noise conditions)

$$F = \left. \frac{S_i/N_i}{S_o/N_o} \right|_{T_0=290K}$$

$$V_{n(ef)} = \sqrt{4kTB R}$$
$$P_n = kTB$$

Noise Figure F



- We identify the two terms:
 - amplified input noise
 - internally generated noise
- When the input noise does not correspond to reference noise conditions ($N_1 \neq N_0$)
 - the internally generated noise does not change

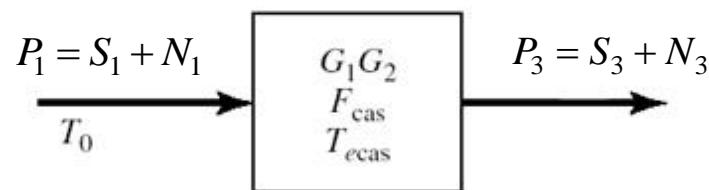
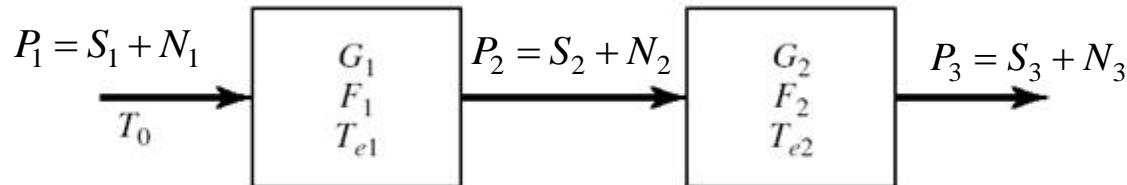
$$N_2 = N_0 \cdot G + (F - 1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F - 1) \cdot N_0 \cdot G$$

↑ ↑

Continue

Noise figure of a cascaded system



$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1$$

$$G_{cas} = G_1 \cdot G_2$$

$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

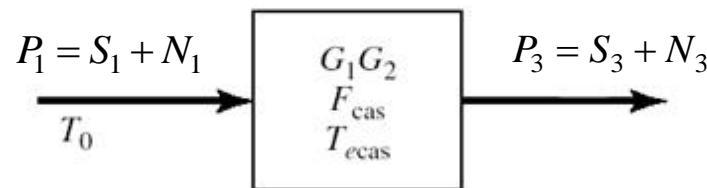
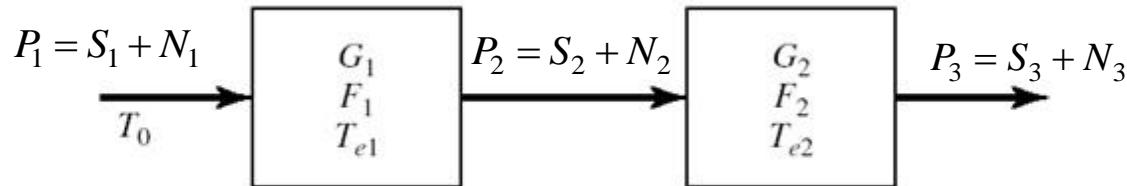
$$N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$



$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

Noise figure of a cascaded system



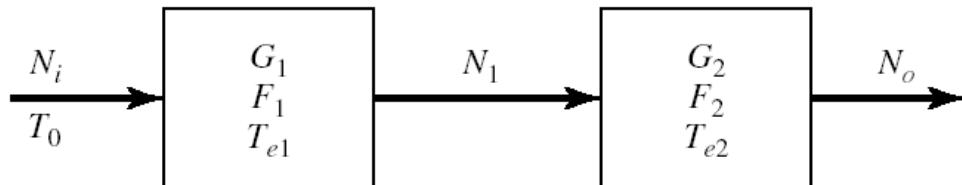
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \quad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

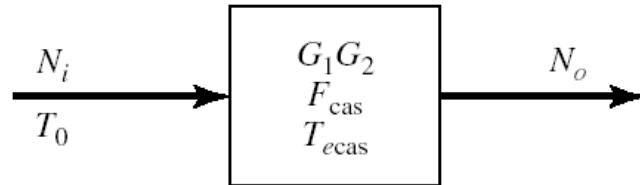
$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1} (F_2 - 1)$$

Noise figure of a cascaded system



(a)



(b)

$$G_{cas} = G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1} (F_2 - 1)$$

- Friis Formula (**!linear scale**)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

- Friis Formula shows that:
 - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
 - the noise introduced by the following stages is reduced:
 - -1
 - division by G (usually $G > 1$)

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \dots$$

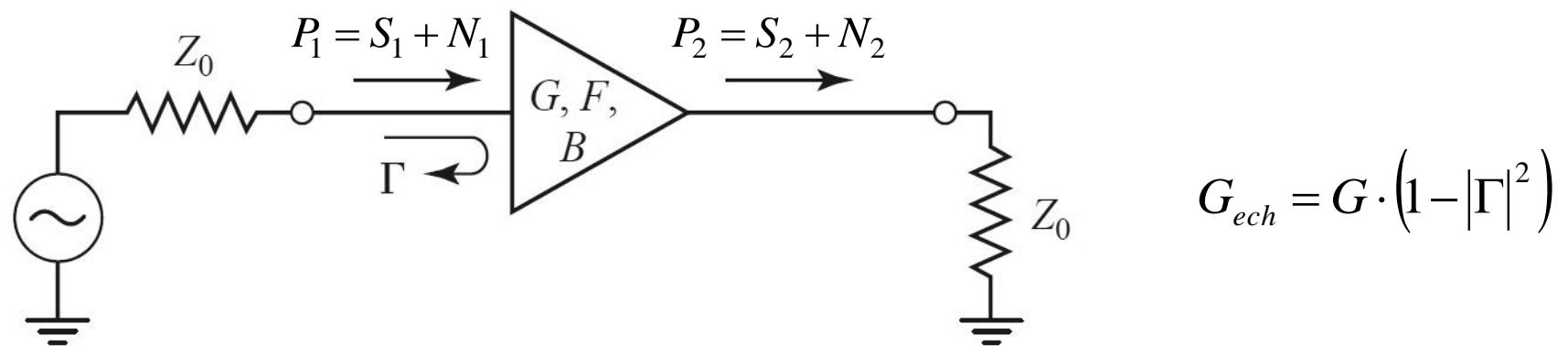
- Effects of Friis Formula:
 - in multi stage amplifiers:
 - it's essential that the first stage is as noiseless as possible even if that means sacrificing power gain
 - the following stages can be optimized for power gain
 - in single stage amplifiers:
 - in the input matching circuit it's important to have noiseless elements (pure reactance, lossless lines)
 - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier ($\Gamma \neq 0$)



$$N_2 = N_1 \cdot G \cdot (1 - |\Gamma|^2) + (F - 1) \cdot N_0 \cdot G = N_1 \cdot G \cdot (1 - |\Gamma|^2) + \frac{F - 1}{1 - |\Gamma|^2} \cdot N_0 \cdot G \cdot (1 - |\Gamma|^2)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech}$$

$$F_{ech} = 1 + \frac{F - 1}{1 - |\Gamma|^2} \geq F$$

- Good noise figure **requires** good impedance matching

Example

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.

- @5GHz

- $S_{11} = 0.64 \angle 139^\circ$
- $S_{12} = 0.119 \angle -21^\circ$
- $S_{21} = 3.165 \angle 16^\circ$
- $S_{22} = 0.22 \angle 146^\circ$
- $F_{min} = 0.54$ (**tipic [dB]**)
- $\Gamma_{opt} = 0.45 \angle 174^\circ$
- $r_n = 0.03$

```
!ATF-34143
IS-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99
```

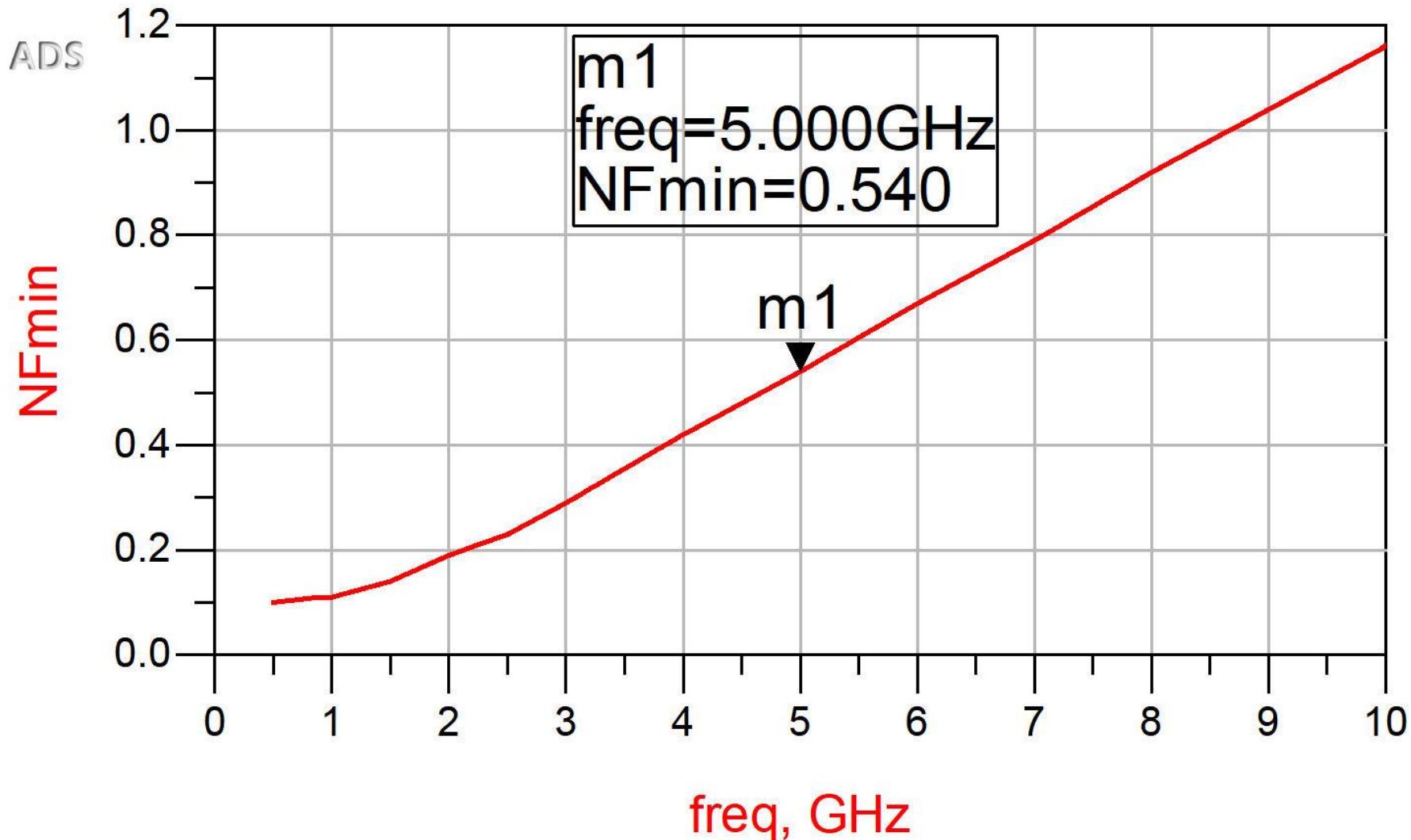
```
# ghz s ma r 50
```

```
2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46
```

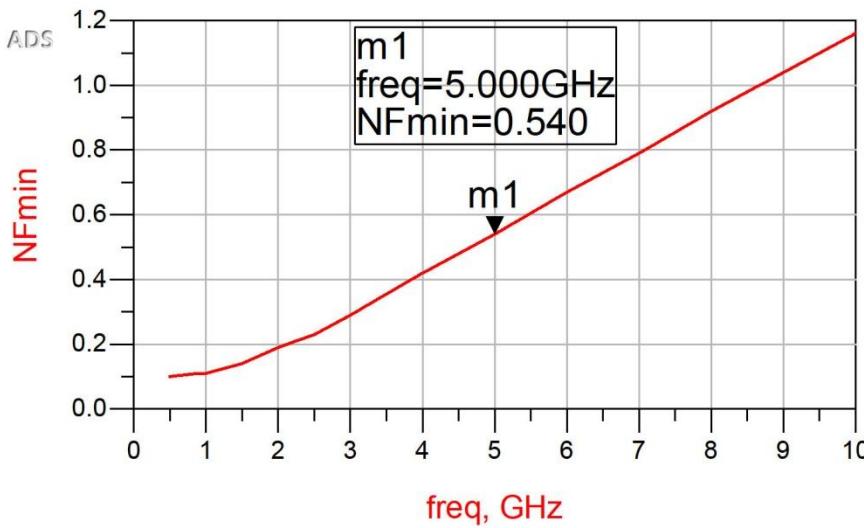
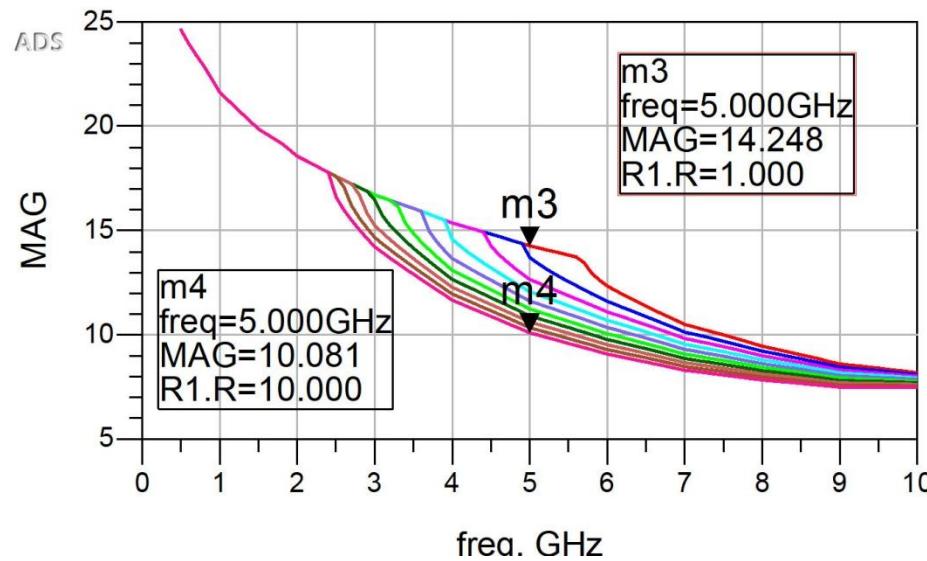
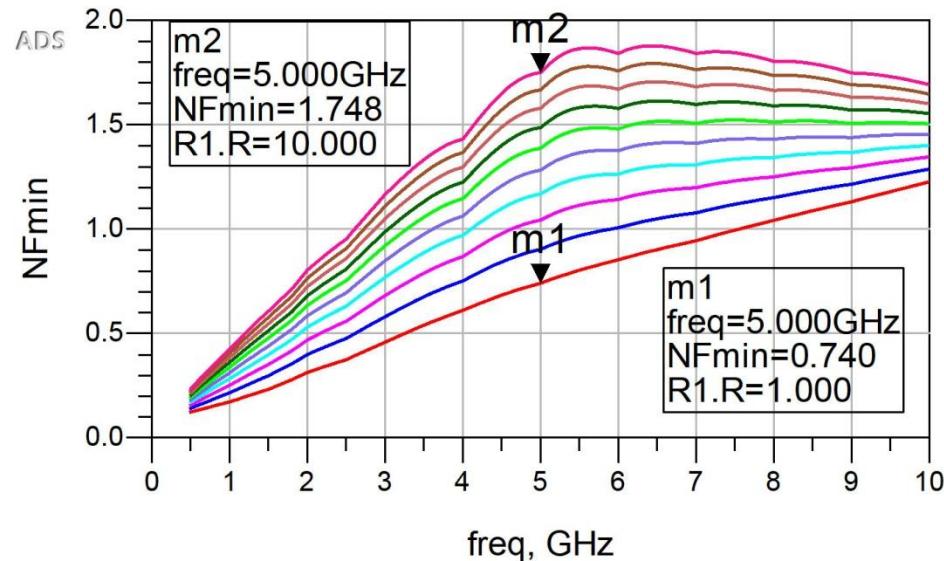
```
!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -
```

```
2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
```

Example

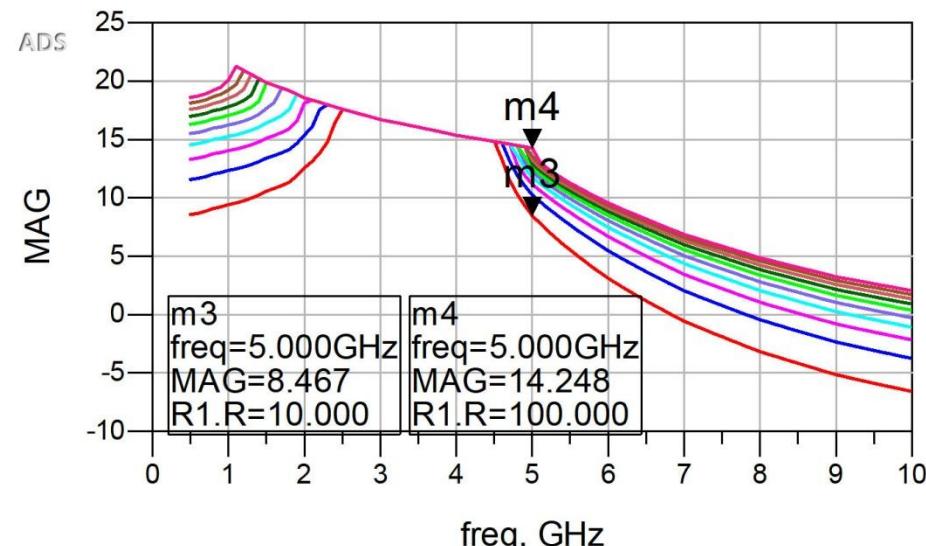
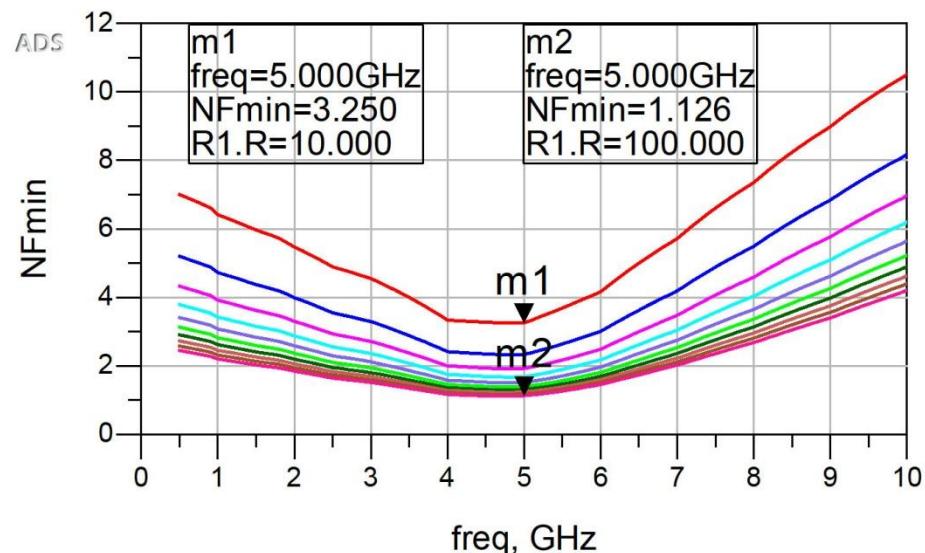


Stabilization, input series resistor

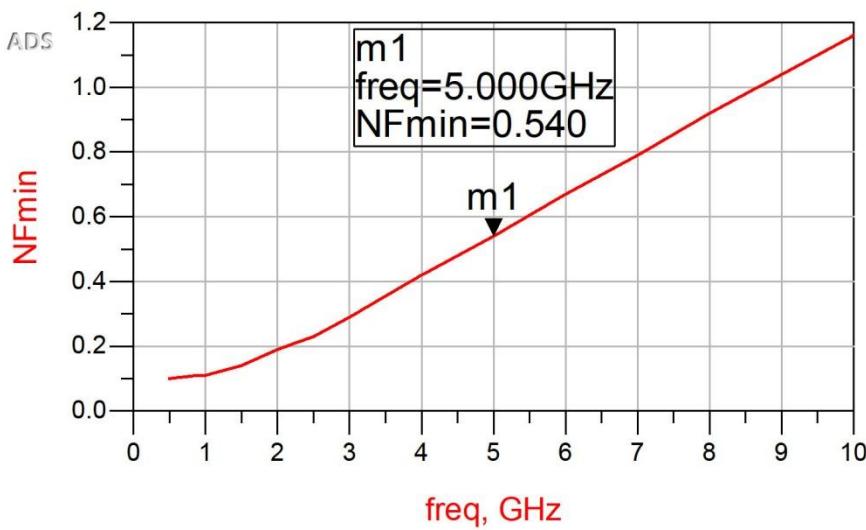


$$R_{SS} = 1 \div 10 \Omega$$

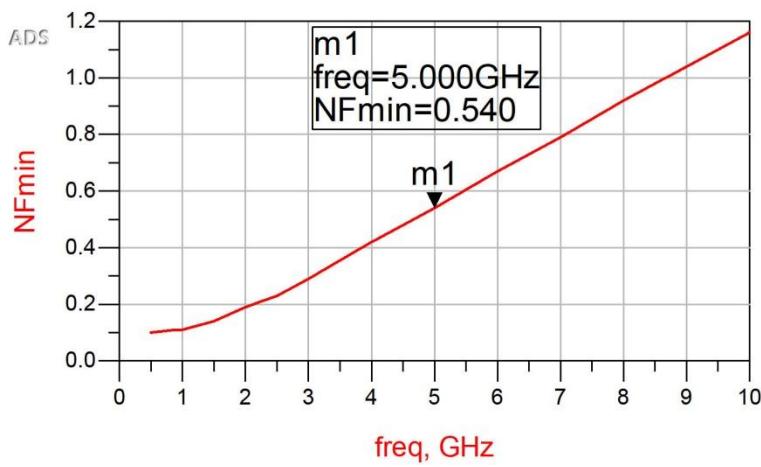
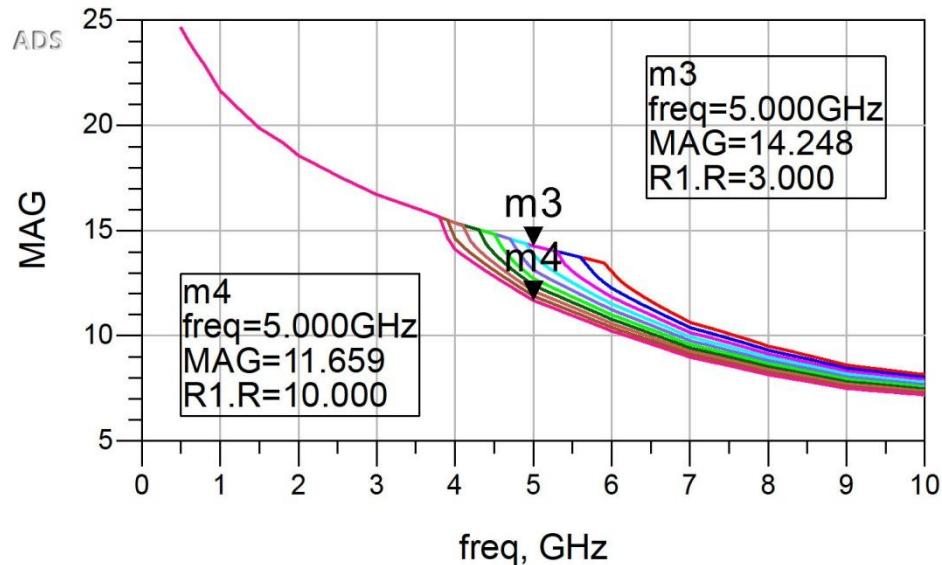
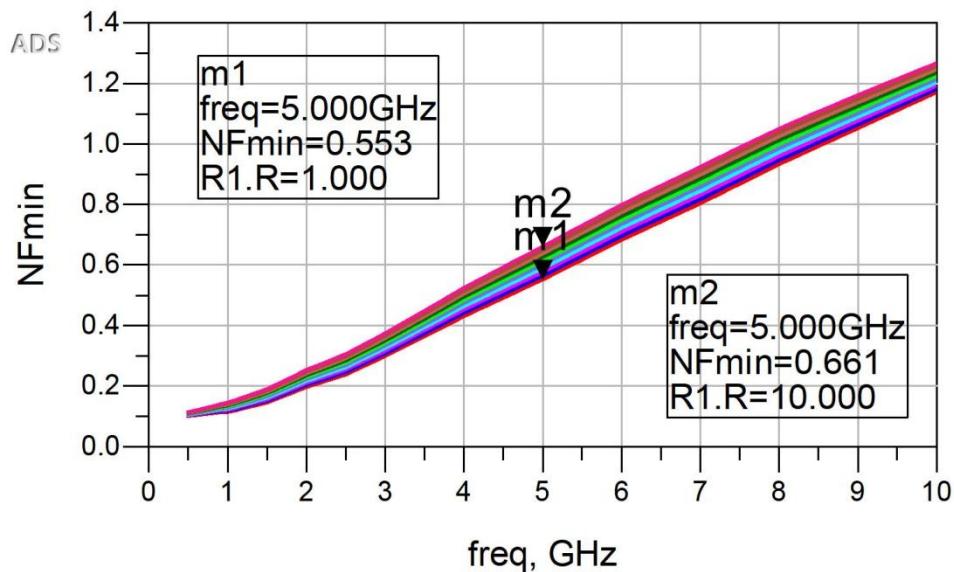
Stabilization, input shunt resistor



$$R_{PS} = 10 \div 100 \Omega$$

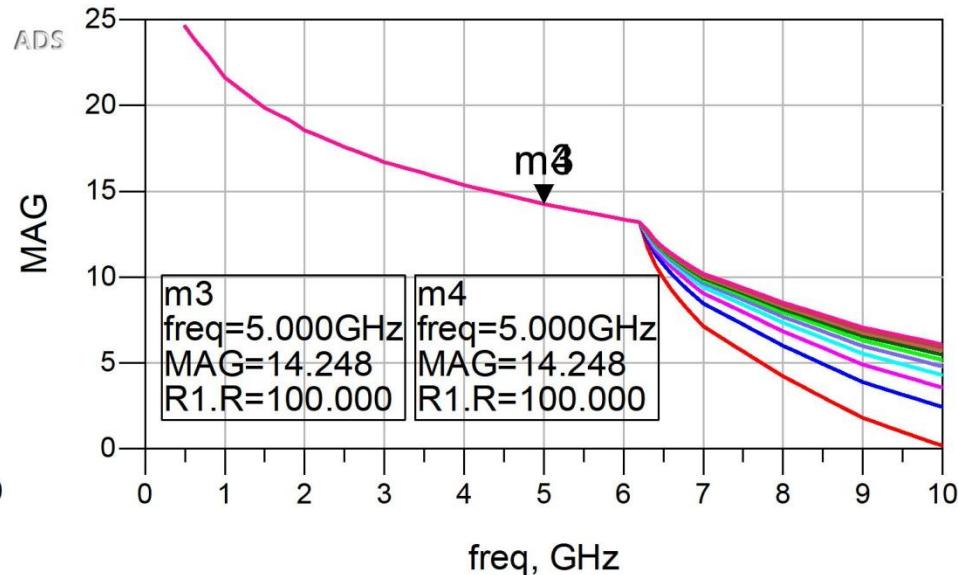
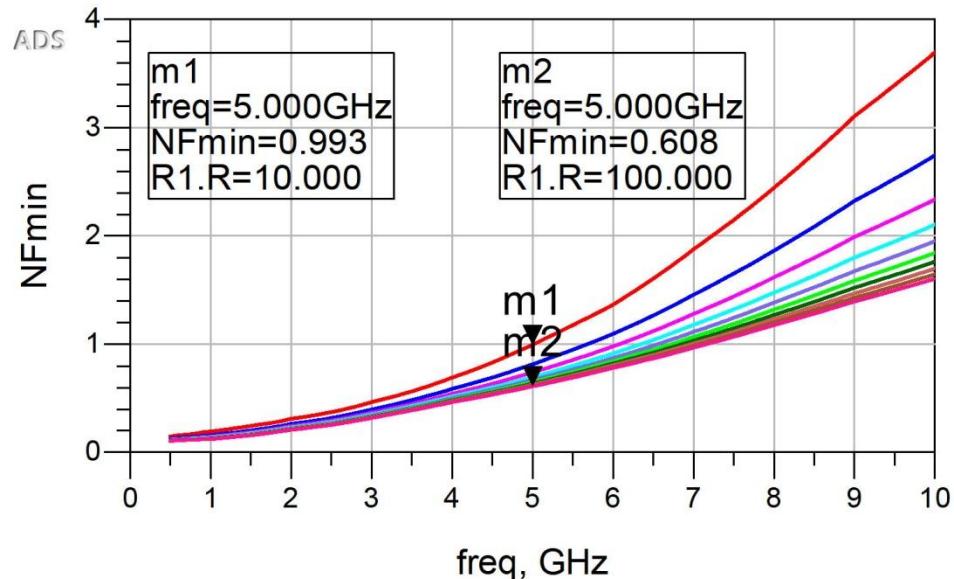


Stabilization, output series resistor

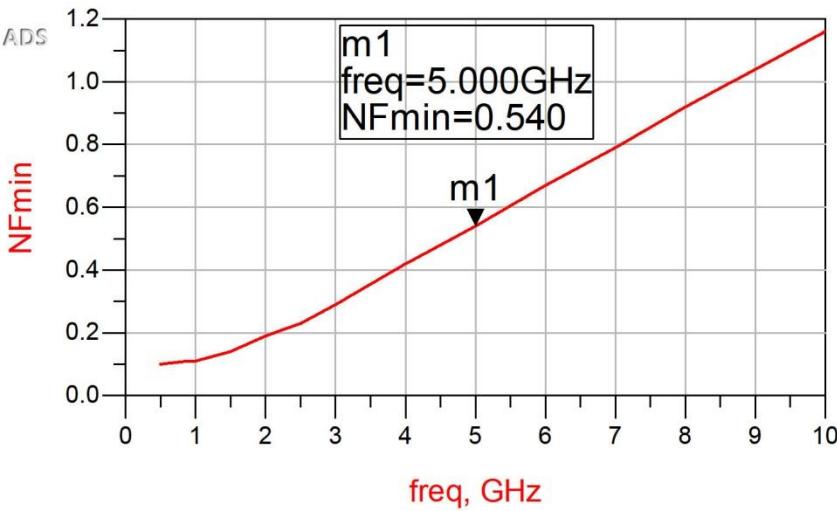


$$R_{SL} = 1 \div 10 \Omega$$

Stabilization, output shunt resistor



$$R_{PL} = 10 \div 100 \Omega$$



Noise figure of a two-port amplifier

- 3 noise parameters (2reals + 1 complex):

$$F_{\min}, r_n = \frac{R_N}{Z_0}, \Gamma_{opt}$$

$$F = F_{\min} + \frac{R_N}{G_S} \cdot |Y_S - Y_{opt}|^2$$

$$Y_S = \frac{1}{Z_0} \cdot \frac{1 - \Gamma_S}{1 + \Gamma_S}$$

$$Y_{opt} = \frac{1}{Z_0} \cdot \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}$$

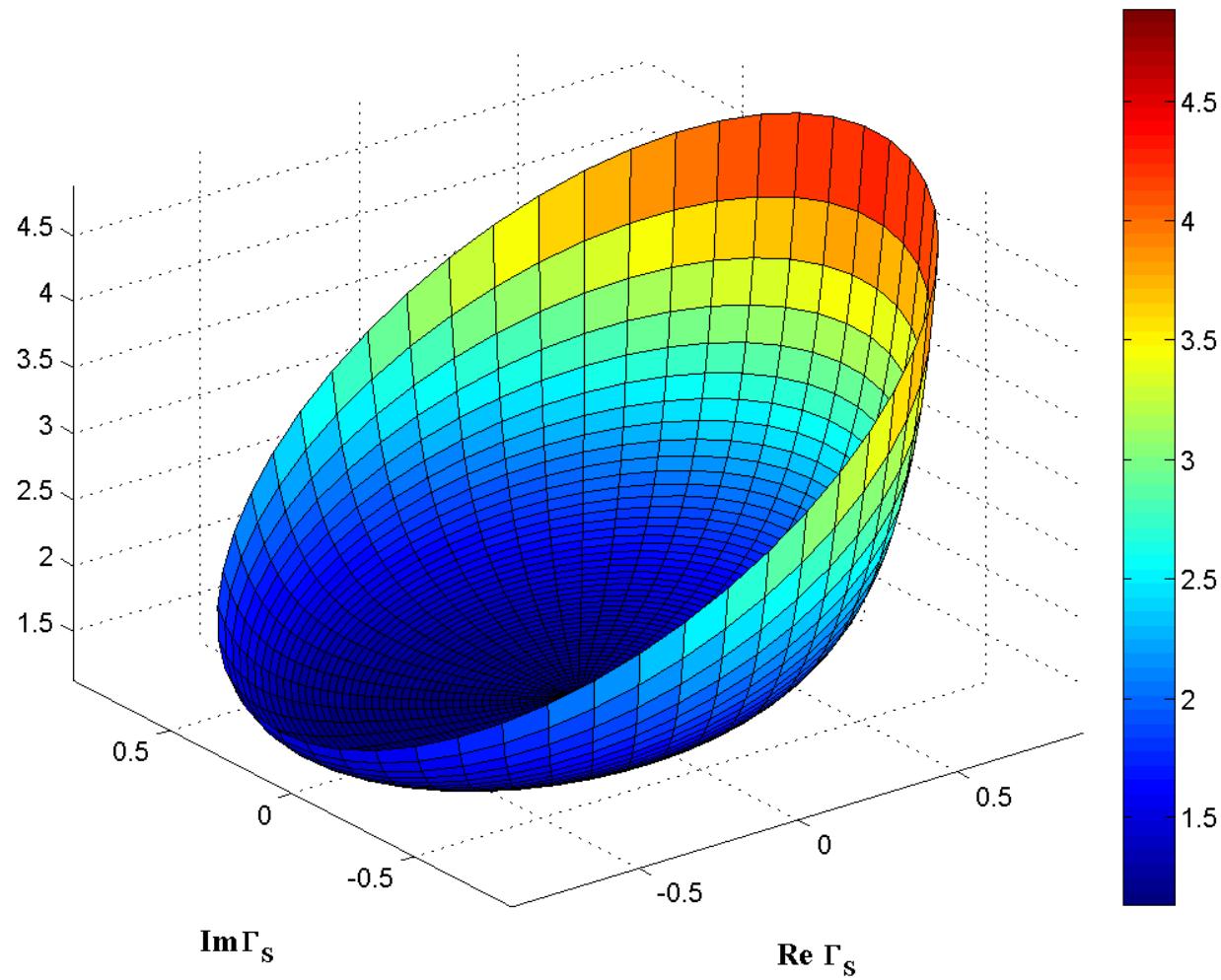
$$F = F_{\min} + 4 \cdot r_n \cdot \frac{|\Gamma_S - \Gamma_{opt}|^2}{(1 - |\Gamma_S|^2) \cdot |1 + \Gamma_{opt}|^2}$$

- Γ_{opt} optimum source reflection coefficient that results in minimum noise figure

$$\Gamma_S = \Gamma_{opt} \Rightarrow F = F_{\min}$$

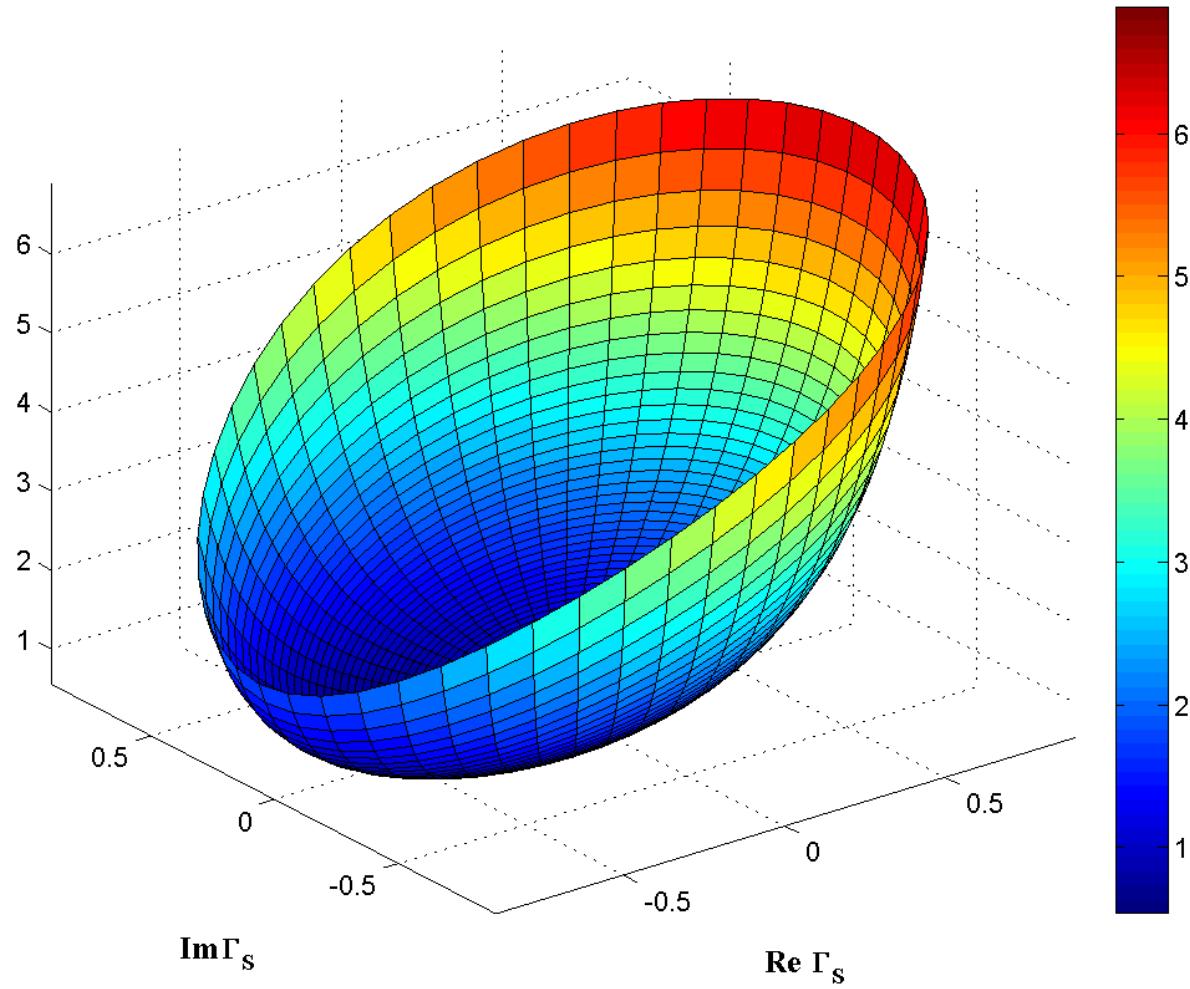
$F(\Gamma_s)$

$F(\Gamma_s)$

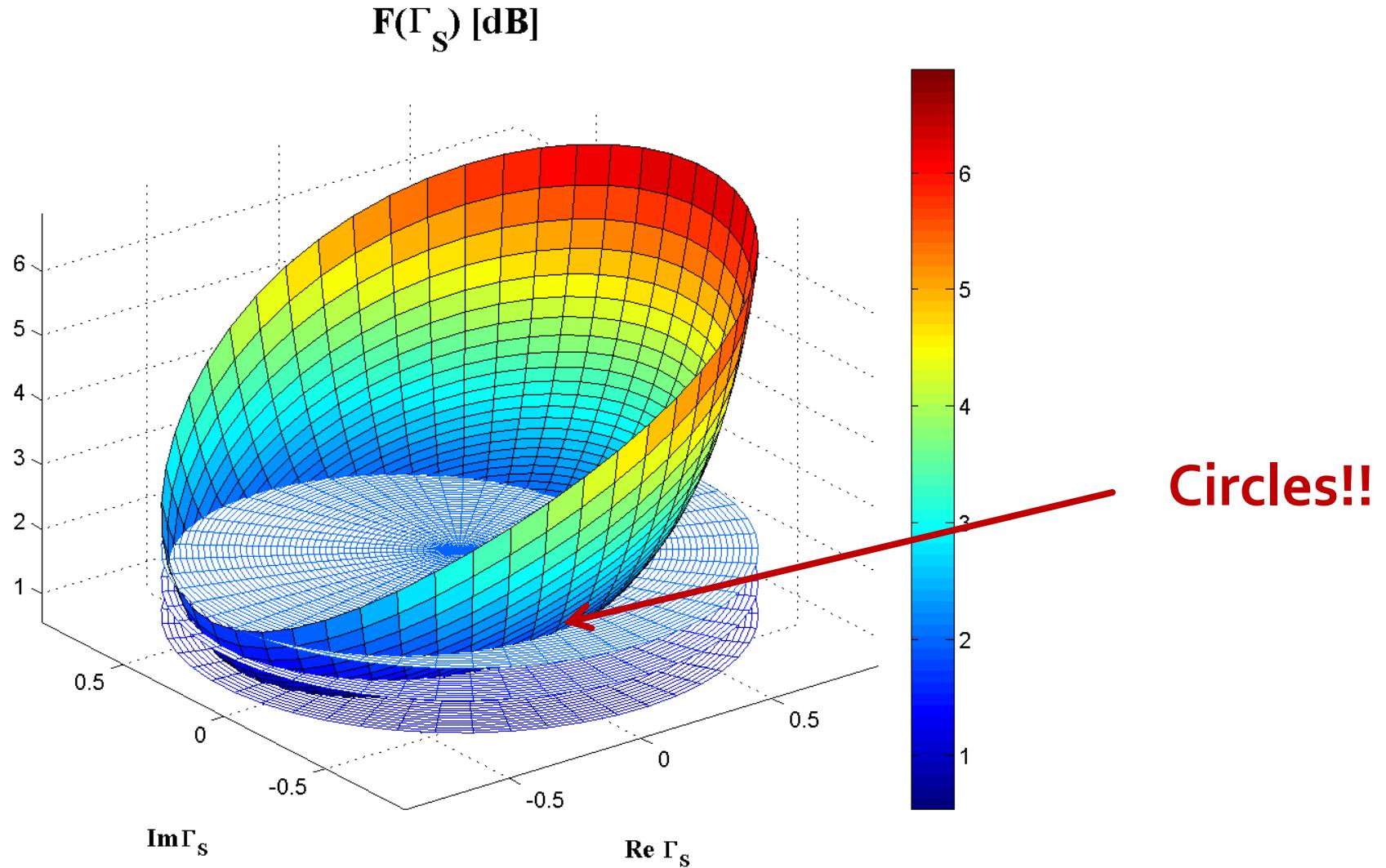


$F[dB](\Gamma_S)$

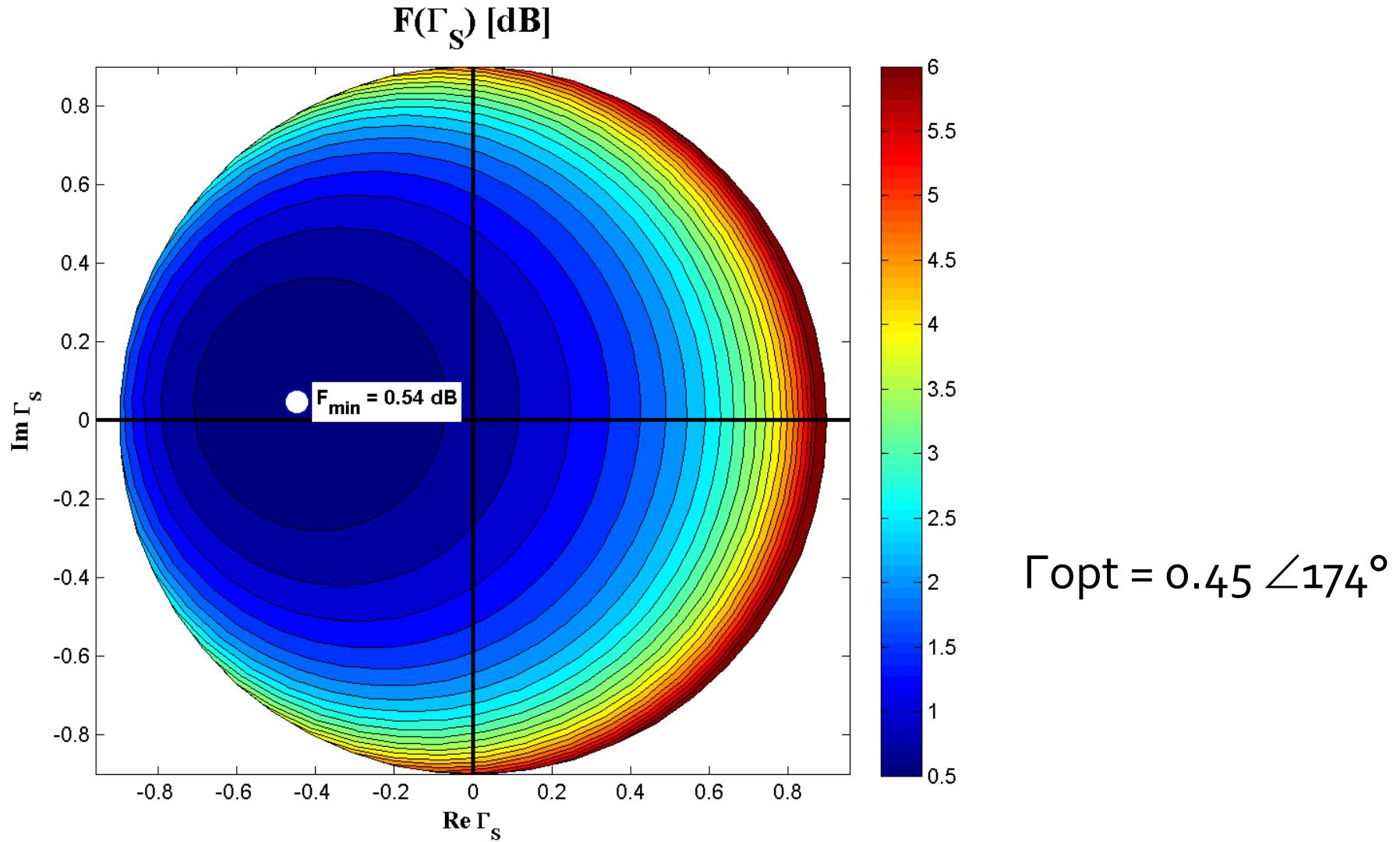
$F(\Gamma_S) [dB]$



$F[\text{dB}](\Gamma_s)$, constant value contours



$G_s[\text{dB}](\Gamma_s)$, constant value contours



Circles of constant noise figure

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{|\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2) \cdot |1 + \Gamma_{opt}|^2}$$

- We define **N** (noise figure parameter)
 - **N** constant for **F** constant

$$N = \frac{|\Gamma_s - \Gamma_{opt}|^2}{1 - |\Gamma_s|^2} = \frac{F - F_{\min}}{4 \cdot r_n} \cdot |1 + \Gamma_{opt}|^2$$

$$(\Gamma_s - \Gamma_{opt}) \cdot (\Gamma_s^* - \Gamma_{opt}^*) = N \cdot (1 - |\Gamma_s|^2)$$

$$\Gamma_s \cdot \Gamma_s^* + N \cdot |\Gamma_s|^2 - (\Gamma_s \cdot \Gamma_{opt}^* - \Gamma_s^* \cdot \Gamma_{opt}) + \Gamma_{opt} \cdot \Gamma_{opt}^* = N$$

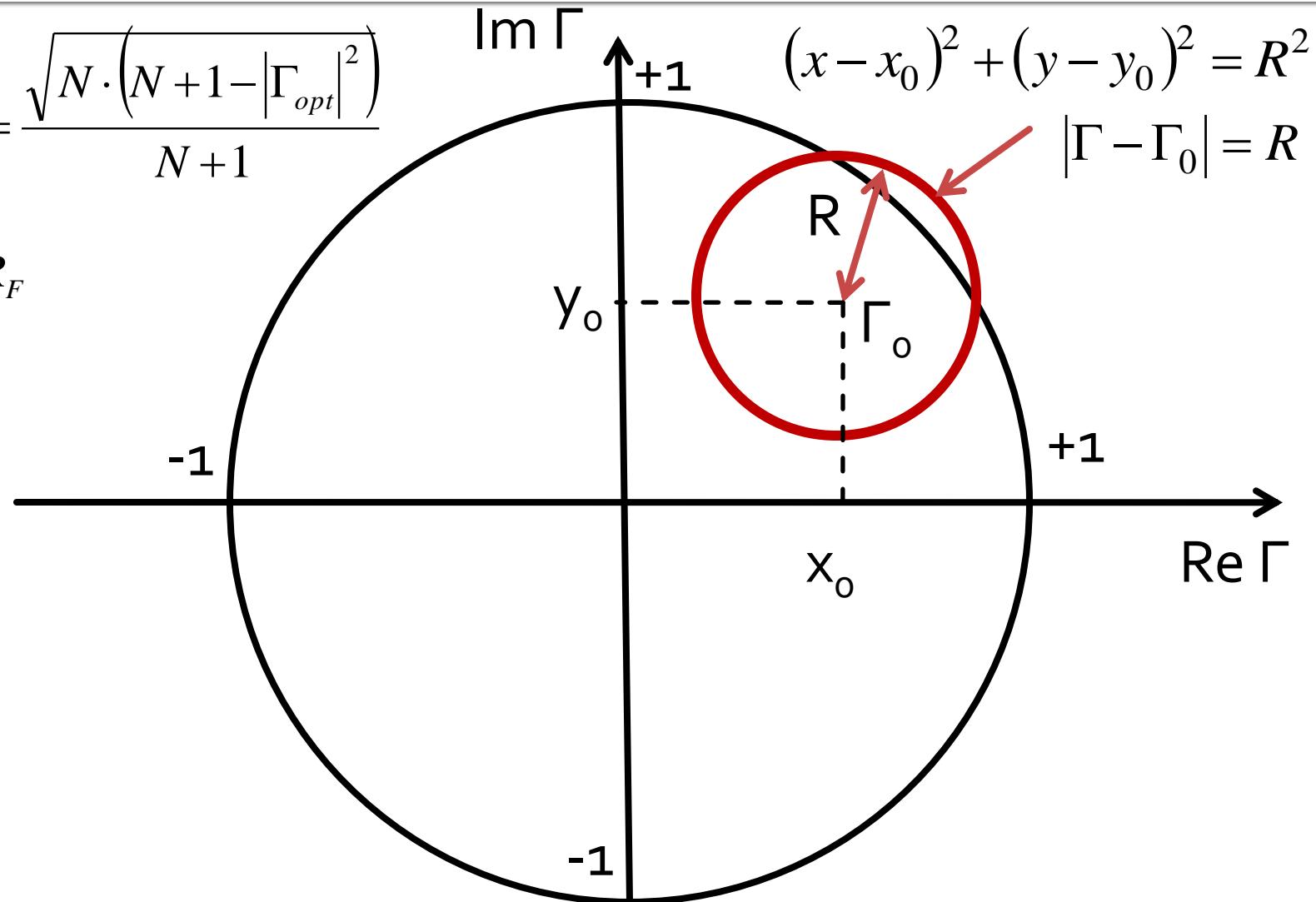
$$\Gamma_s \cdot \Gamma_s^* - \frac{\Gamma_s \cdot \Gamma_{opt}^* - \Gamma_s^* \cdot \Gamma_{opt}}{N+1} + \Gamma_{opt} \cdot \Gamma_{opt}^* = \frac{N - |\Gamma_{opt}|^2}{N+1} + \frac{|\Gamma_{opt}|^2}{(N+1)^2}$$

$$|a+b|^2 = (a+b) \cdot (a+b)^* = (a+b) \cdot (a^* + b^*) = \underline{|a|^2 + |b|^2} + \underline{a^* \cdot b} + \underline{a \cdot b^*}$$

Circles of constant noise figure

$$\left| \Gamma_s - \frac{\Gamma_{opt}}{N+1} \right| = \sqrt{N \cdot \left(N + 1 - |\Gamma_{opt}|^2 \right)} / (N+1)$$

$$|\Gamma_s - C_F| = R_F$$



Circles of constant noise figure

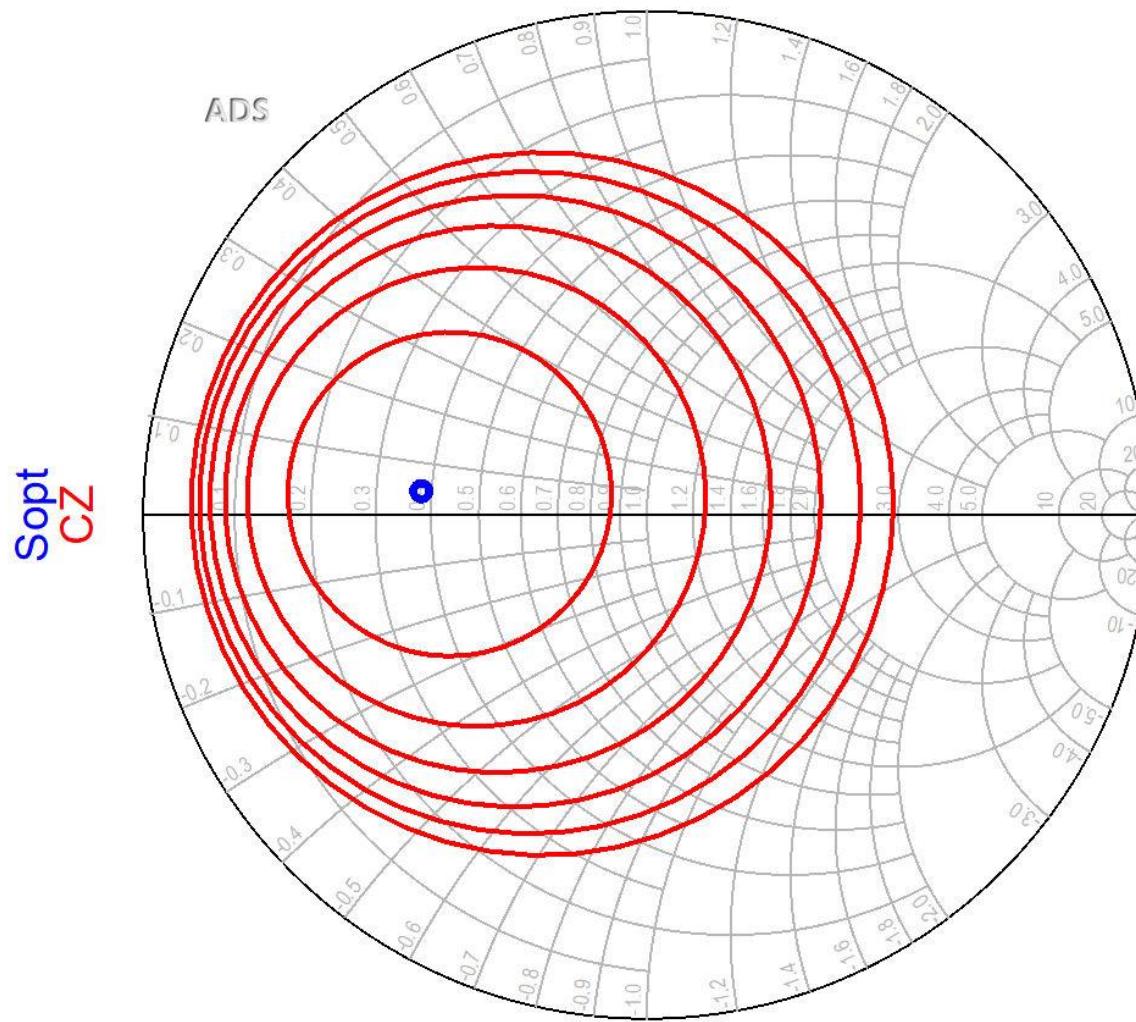
$$\left| \Gamma_S - \frac{\Gamma_{opt}}{N+1} \right| = \frac{\sqrt{N \cdot (N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

$$N = \frac{F - F_{\min}}{4 \cdot r_n} \cdot |1 + \Gamma_{opt}|^2$$

$$|\Gamma_S - C_F| = R_F \quad C_F = \frac{\Gamma_{opt}}{N+1} \quad R_F = \frac{\sqrt{N \cdot (N+1 - |\Gamma_{opt}|^2)}}{N+1}$$

- The locus in the complex plane Γ_S of the points with constant noise figure is a circle
- **Interpretation:** Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for F_{circle} will lead to a noise factor $F = F_{\text{circle}}$
 - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a noise factor $F > F_{\text{circle}}$
 - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a noise factor $F < F_{\text{circle}}$

ADS

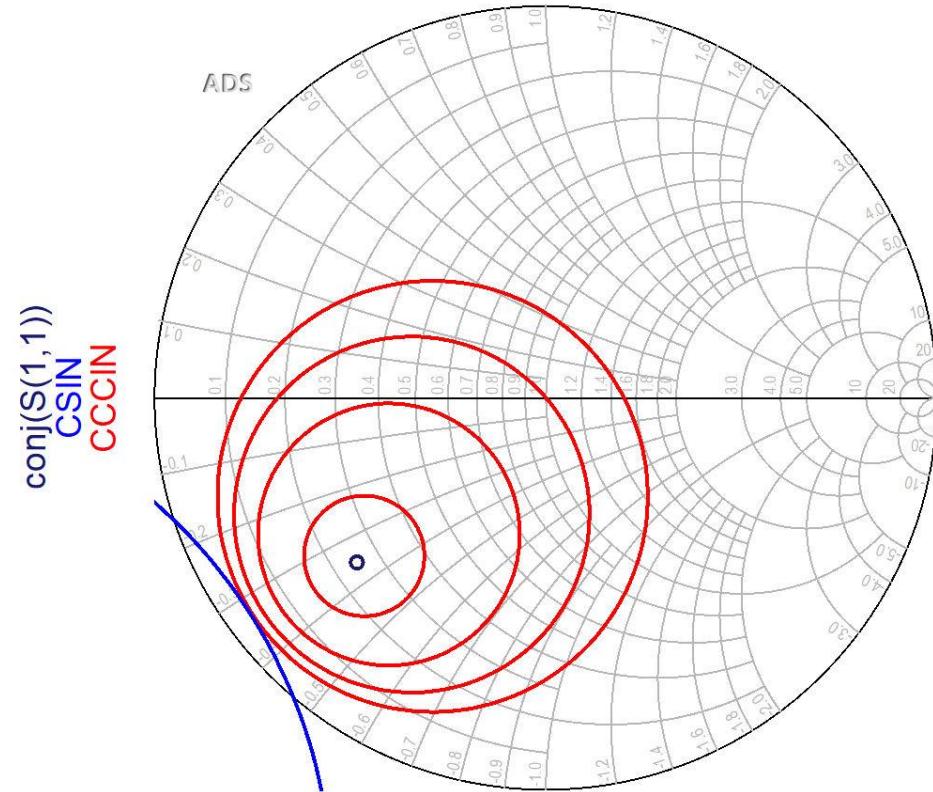
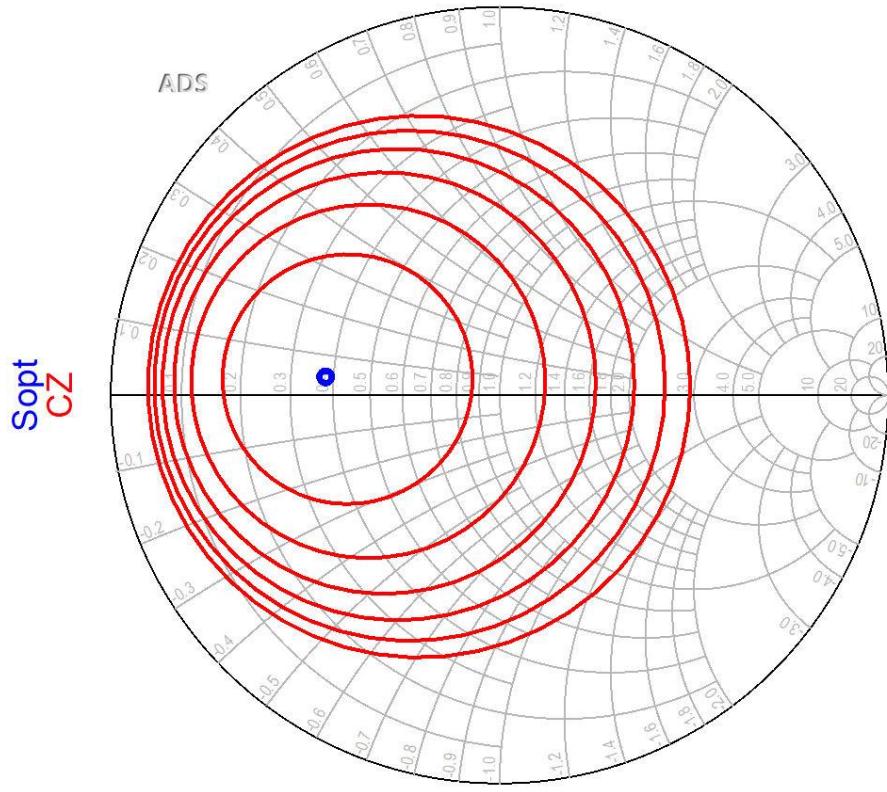


Circles of constant noise figure

- The noise internally generated by the transistor depends **only** by the input matching circuit
- A minimum noise figure is possible (NF_{min} – a datasheet/"s2p file" parameter for the transistor)
- If we design a low noise amplifier (**LNA**) the usual design technique is as follows:
 - design of the input matching circuit solely (largely) for noise optimization
 - design of output matching circuit for gain compensation/optimization (if lossy circuits are used the output matching circuit noise can be added but the transistor noise is not influenced)

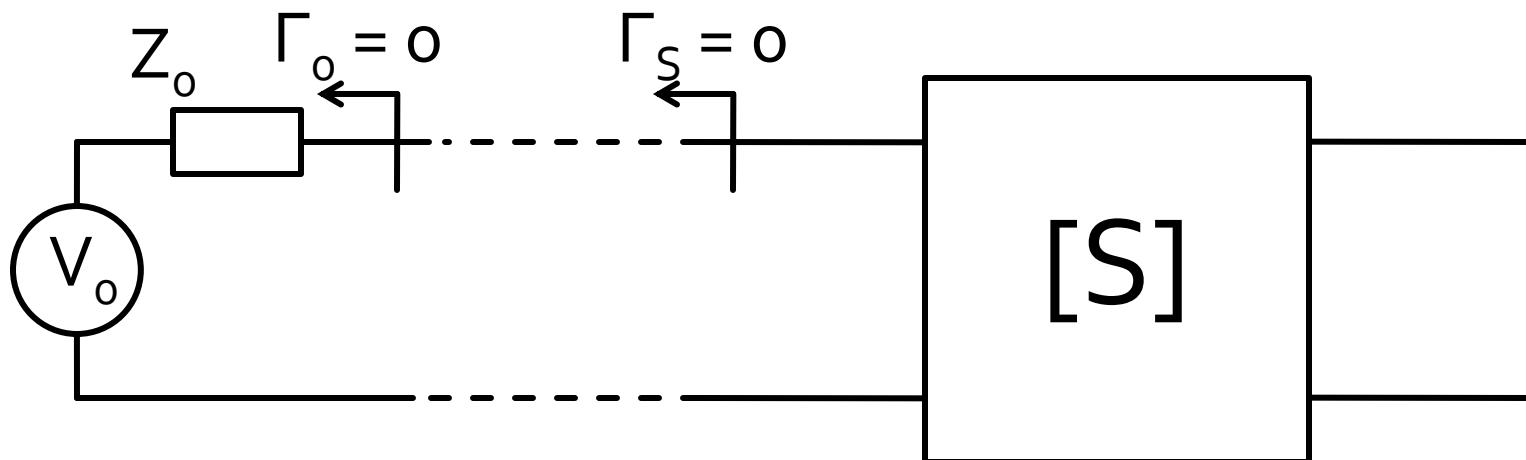
LNA – Low Noise Amplifier

- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for Γ_s



Matching – 1

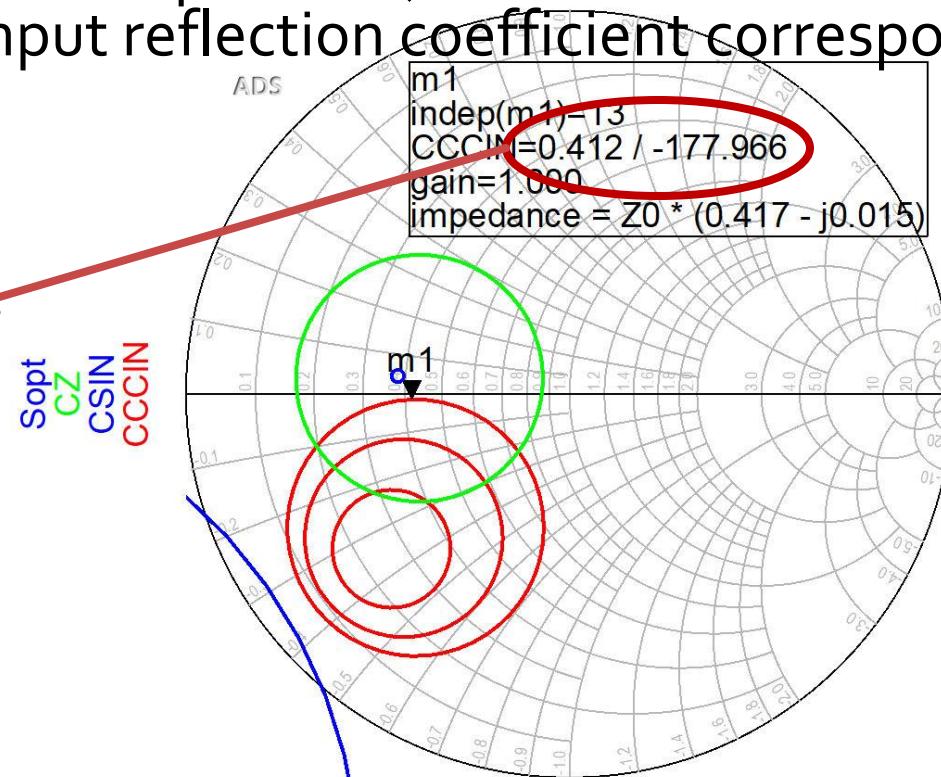
- Connecting the amplifier (transistor) directly to the source with Z_o generate a reflection coefficient seen towards the source equal with **0** (complex number, $\Gamma_o = 0 + 0 \cdot j$)
 - most of the time this reflection coefficient does not offer optimum noise/gain



Matching – 2

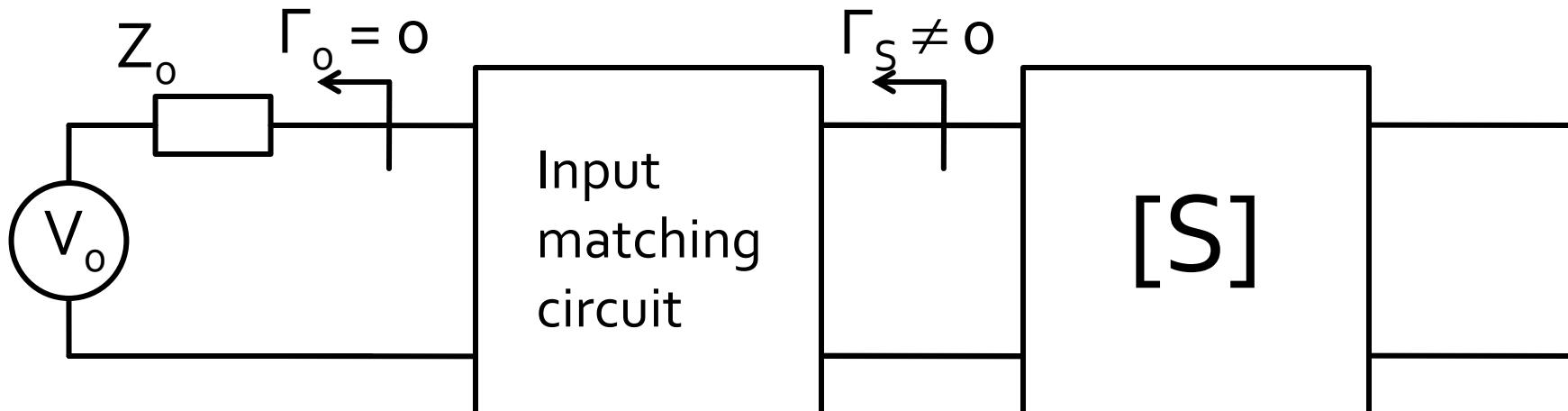
- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point, Γ_s

$$\Gamma_s = 0.412 \angle -177.966^\circ$$



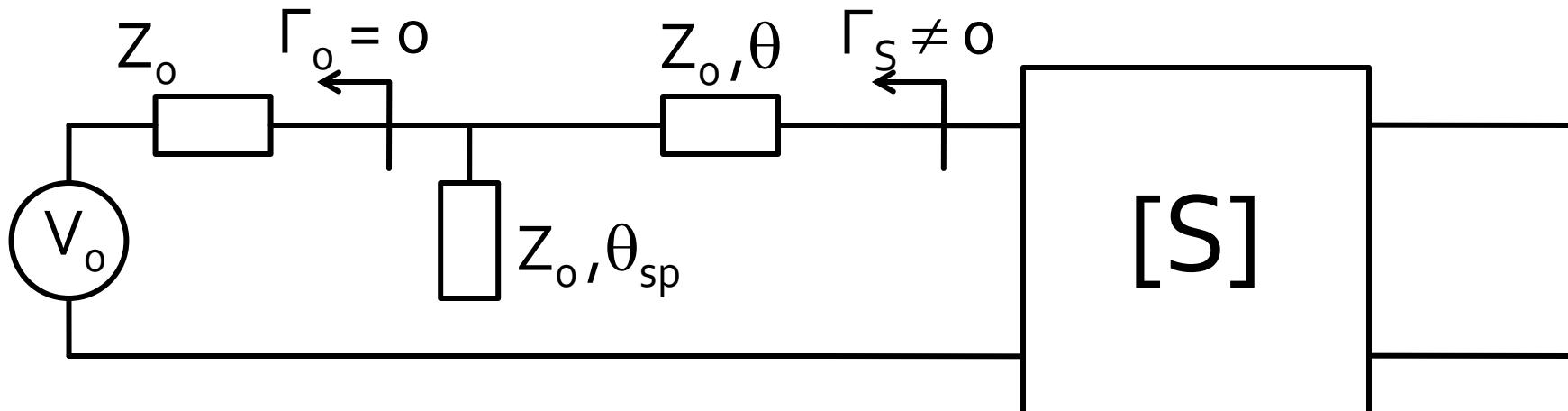
Matching – 3

- We insert the input matching circuits which allows the transistor to see towards the source the previously determined reflection coefficient Γ_s



Matching – 4

- Easiest to design matching section consists in the insertion of (in order from the transistor towards the Z_o source):
 - a series Z_o line, with electrical length θ
 - a shunt stub, open-circuited, made from a Z_o line, with electrical length θ_{sp}



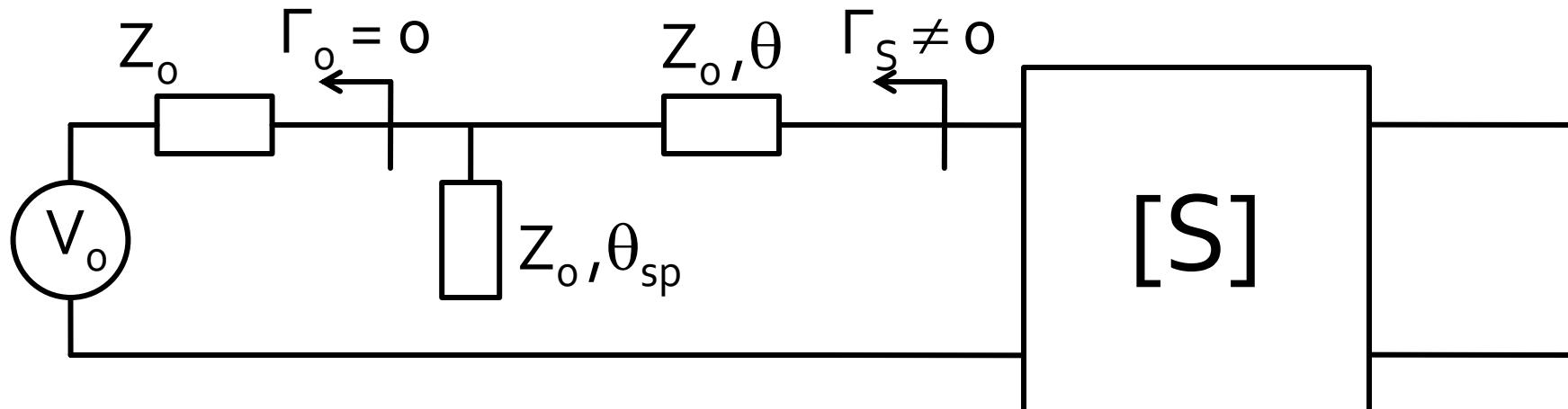
Matching – 5

- Computation depends solely on Γ_s (magnitude and phase)

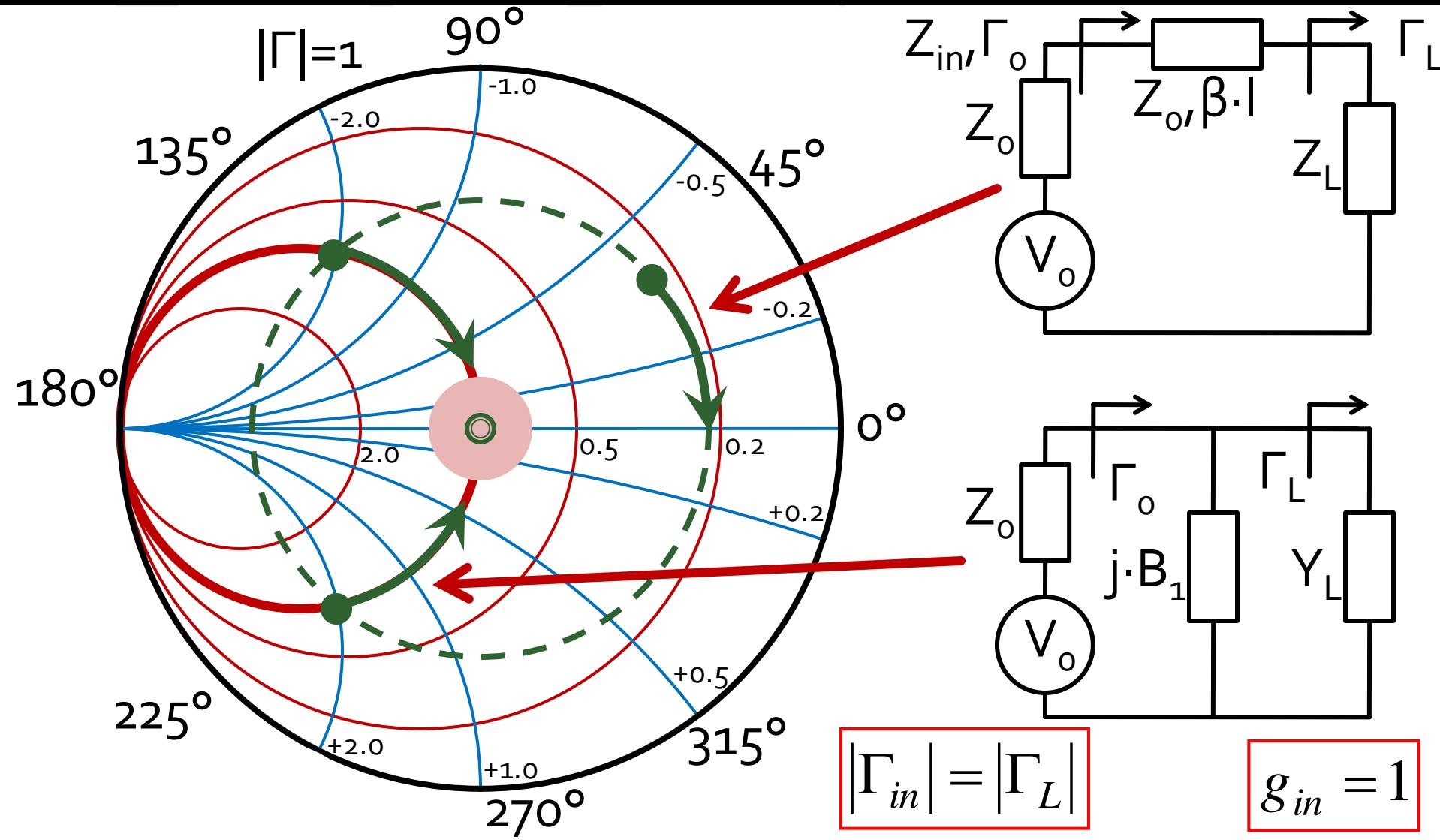
$$\cos(\varphi_s + 2\theta) = -|\Gamma_s|$$

$$\tan \theta_{sp} = \frac{\mp 2 \cdot |\Gamma_s|}{\sqrt{1 - |\Gamma_s|^2}}$$

- The sign (+/-) chosen for the series line equation imposes the sign used for the shunt stub equation



Shunt stub matching, L₇



Example, LNA @ 5 GHz

- ATF-34143 at $V_{ds}=3V$ $I_d=20mA$.

- @5GHz

- $S_{11} = 0.64 \angle 139^\circ$
- $S_{12} = 0.119 \angle -21^\circ$
- $S_{21} = 3.165 \angle 16^\circ$
- $S_{22} = 0.22 \angle 146^\circ$
- $F_{min} = 0.54$ (**tipic [dB]**)
- $\Gamma_{opt} = 0.45 \angle 174^\circ$
- $r_n = 0.03$

```
!ATF-34143
IS-PARAMETERS at Vds=3V Id=20mA. LAST UPDATED 01-29-99
```

```
# ghz s ma r 50
```

```
2.0 0.75 -126 6.306 90 0.088 23 0.26 -120
2.5 0.72 -145 5.438 75 0.095 15 0.25 -140
3.0 0.69 -162 4.762 62 0.102 7 0.23 -156
4.0 0.65 166 3.806 38 0.111 -8 0.22 174
5.0 0.64 139 3.165 16 0.119 -21 0.22 146
6.0 0.65 114 2.706 -5 0.125 -35 0.23 118
7.0 0.66 89 2.326 -27 0.129 -49 0.25 91
8.0 0.69 67 2.017 -47 0.133 -62 0.29 67
9.0 0.72 48 1.758 -66 0.135 -75 0.34 46
```

```
!FREQ Fopt GAMMA OPT RN/Zo
!GHZ dB MAG ANG -
```

```
2.0 0.19 0.71 66 0.09
2.5 0.23 0.65 83 0.07
3.0 0.29 0.59 102 0.06
4.0 0.42 0.51 138 0.03
5.0 0.54 0.45 174 0.03
6.0 0.67 0.42 -151 0.05
7.0 0.79 0.42 -118 0.10
8.0 0.92 0.45 -88 0.18
9.0 1.04 0.51 -63 0.30
10.0 1.16 0.61 -43 0.46
```

Example, LNA @ 5 GHz

- Low Noise Amplifier
- At the input matching a compromise is required between:
 - noise (**input** constant noise figure circles)
 - gain (input constant gain circles)
 - stability (input stability circle)
- At the output matching noise **is not influenced**.
A compromise is required between :
 - gain (output constant gain circles)
 - stability (output stability circle)

Example, LNA @ 5 GHz

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{(1 - |S_{11}|^2) \cdot (1 - |S_{22}|^2)} = 0.094 \quad -0.783 \text{ dB} < G_T[\text{dB}] - G_{TU}[\text{dB}] < 0.861 \text{ dB}$$

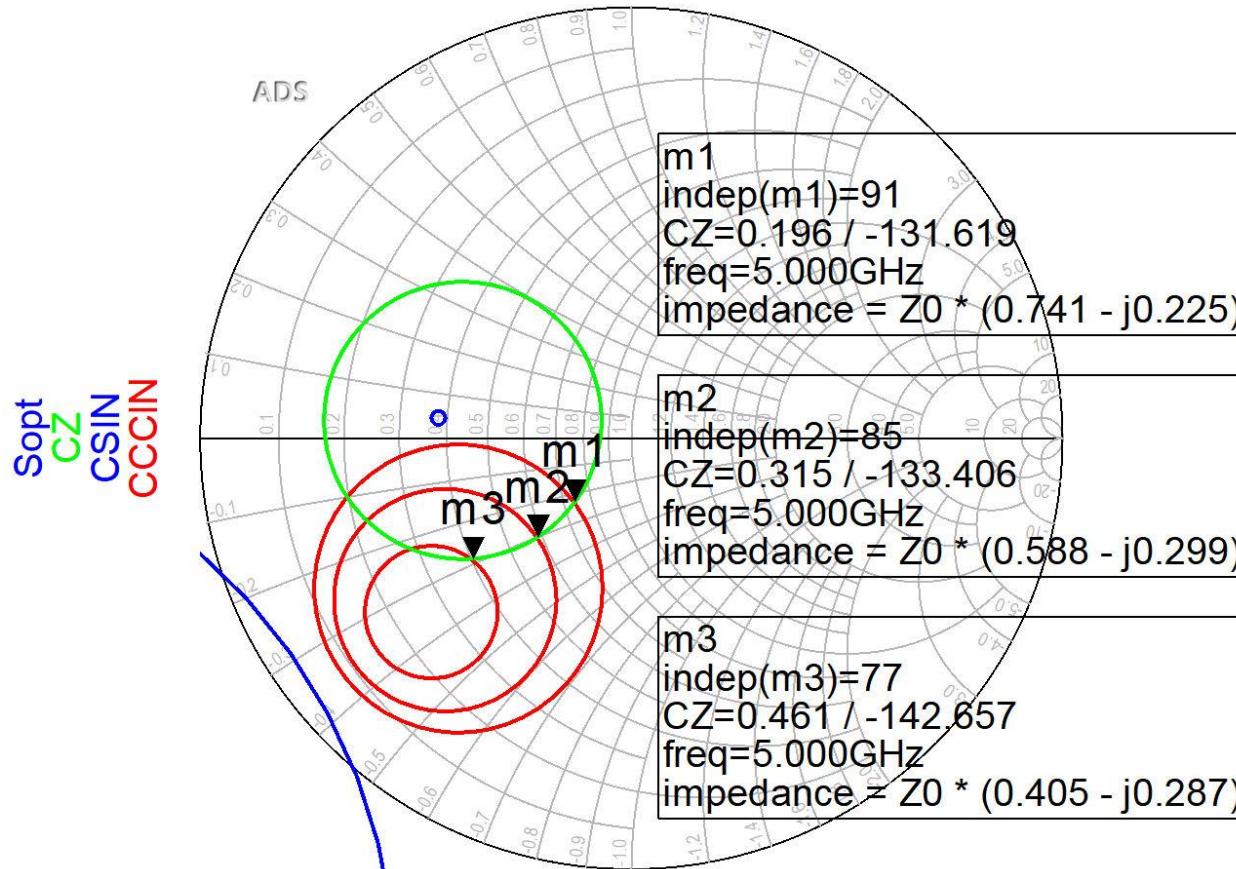
$$G_{TU\max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 17.83 \quad G_{TU\max} [\text{dB}] = 12.511 \text{ dB}$$

$$G_0 = |S_{21}|^2 = 10.017 = 10.007 \text{ dB}$$

$$G_{S\max} = \frac{1}{1 - |S_{11}|^2} = 1.694 = 2.289 \text{ dB} \quad G_{L\max} = \frac{1}{1 - |S_{22}|^2} = 1.051 = 0.215 \text{ dB}$$

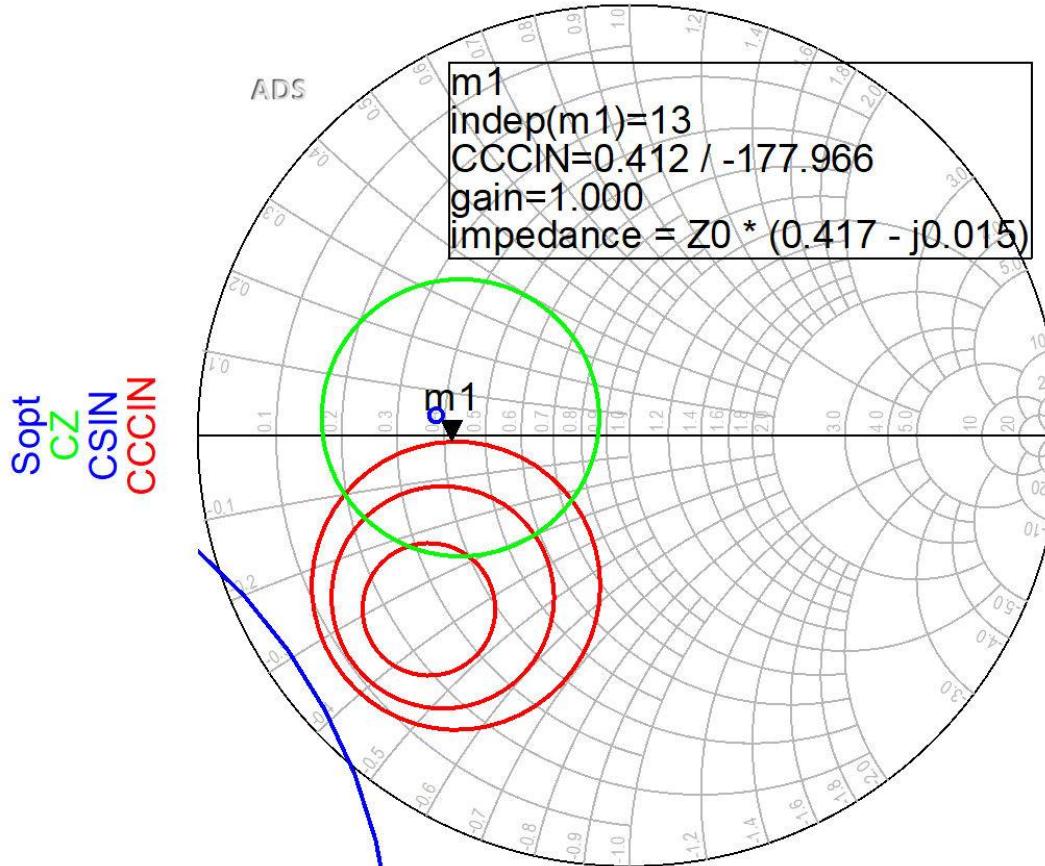
- In this particular case $G_{L\max} = 0.21 \text{ dB}$, the transistor could be used directly connected to the 50Ω load
- The absence of the output matching circuit **is not** recommended. While the attainable power gain is low, its absence eliminates the possibility to use it to compensate an improper gain generated by the noise optimization of the input matching circuit

Input matching circuit



- For the input matching circuit
 - noise circle CZ : 0.75dB
 - input constant gain circles $CCCIN$: 1dB, 1.5dB, 2 dB
- We choose (small $Q \rightarrow$ wide bandwidth) position $m1$

Input matching circuit



- If we can afford a 1.2dB decrease of the input gain for better NF,Q ($G_s = 1$ dB), position *m1* above is better
- We obtain better (smaller) NF

Input matching circuit

- Position m1 in complex plane (Smith Chart)

$$\Gamma_S = 0.412 \angle -178^\circ$$

$$|\Gamma_S| = 0.412; \quad \varphi = -178^\circ$$

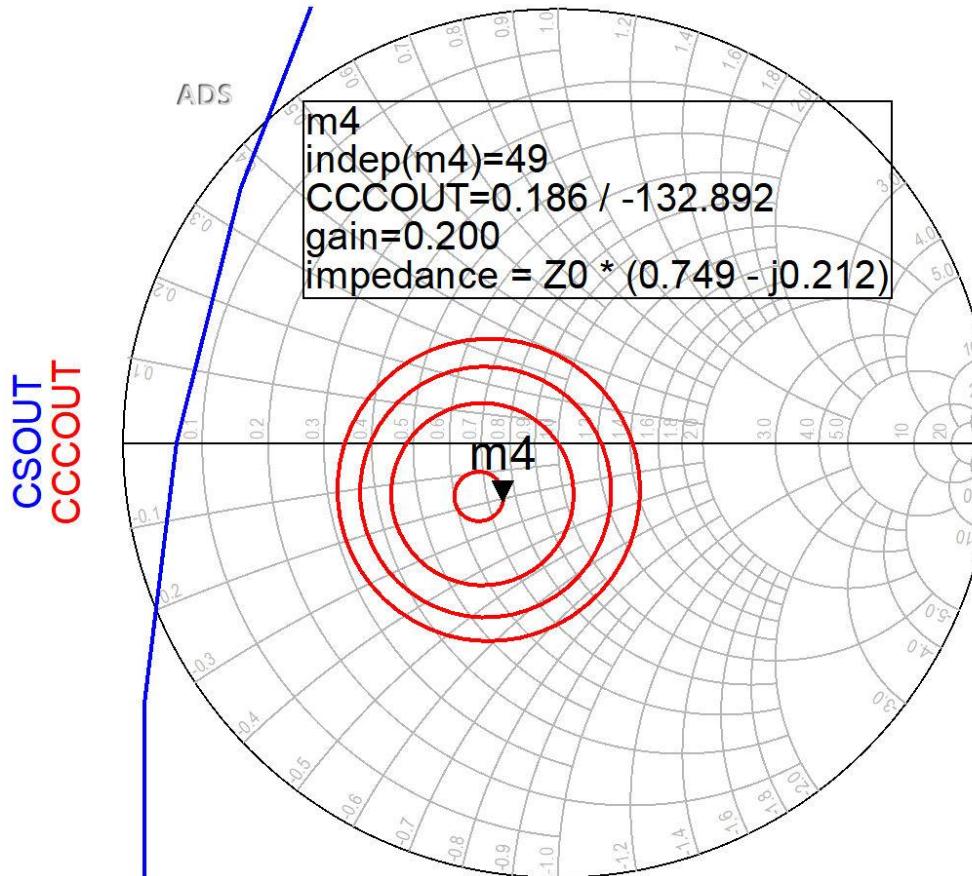
$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\text{Im}[y_S(\theta)] = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\cos(\varphi + 2\theta) = -0.412 \Rightarrow (\varphi + 2\theta) = \pm 114.33^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +114.33^\circ \\ -114.33^\circ \end{cases} \quad \theta = \begin{cases} 146.2^\circ \\ 31.8^\circ \end{cases} \quad \text{Im}[y_S(\theta)] = \begin{cases} -0.904 \\ +0.904 \end{cases} \quad \theta_{sp} = \begin{cases} 137.9^\circ \\ 42.1^\circ \end{cases}$$

Output matching circuit



- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- the lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

Output matching circuit

- Position m₄ in complex plane (Smith Chart)

$$\Gamma_L = 0.186 \angle -132.9^\circ$$

$$|\Gamma_L| = 0.186; \quad \varphi = -132.9^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\text{Im}[y_L(\theta)] = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -0.379$$

$$\cos(\varphi + 2\theta) = -0.186 \Rightarrow (\varphi + 2\theta) = \pm 100.72^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +100.72^\circ \\ -100.72^\circ \end{cases} \quad \theta = \begin{cases} 116.8^\circ \\ 16.1^\circ \end{cases} \quad \text{Im}[y_L(\theta)] = \begin{cases} -0.379 \\ +0.379 \end{cases} \quad \theta_{sp} = \begin{cases} 159.3^\circ \\ 20.7^\circ \end{cases}$$

LNA

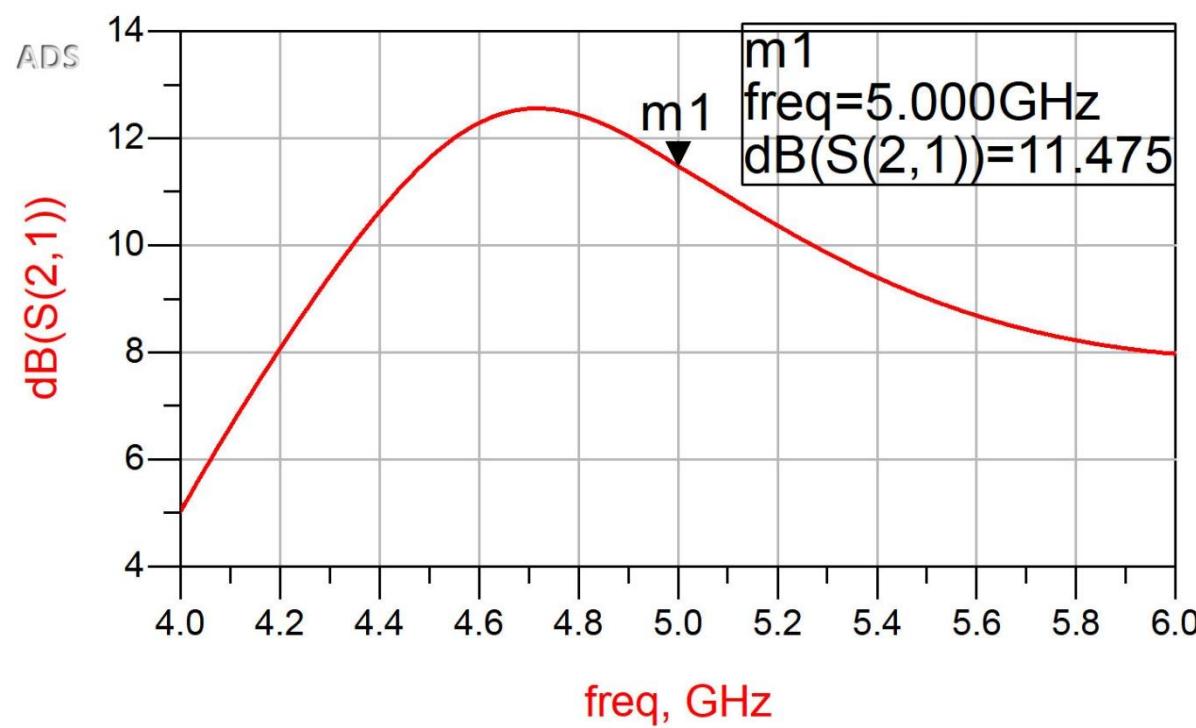
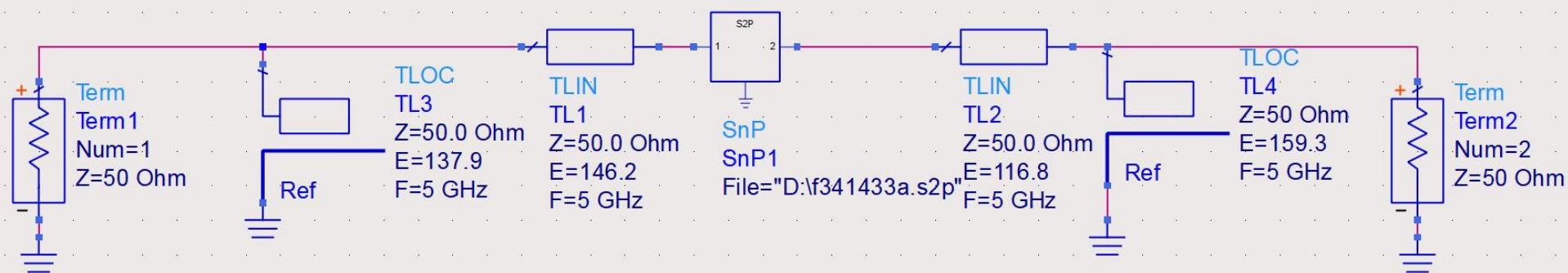
- We estimate a gain (in unilateral assumption, ± 0.9 dB)

$$G_T[\text{dB}] = G_S[\text{dB}] + G_0[\text{dB}] + G_L[\text{dB}]$$

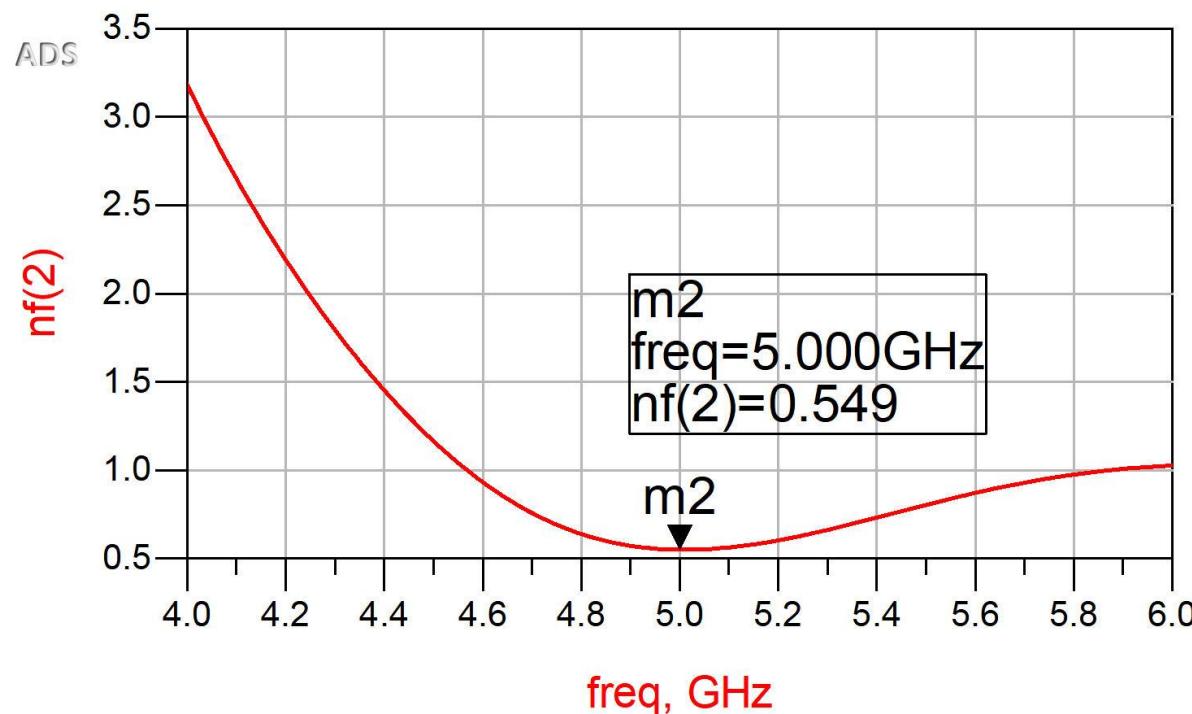
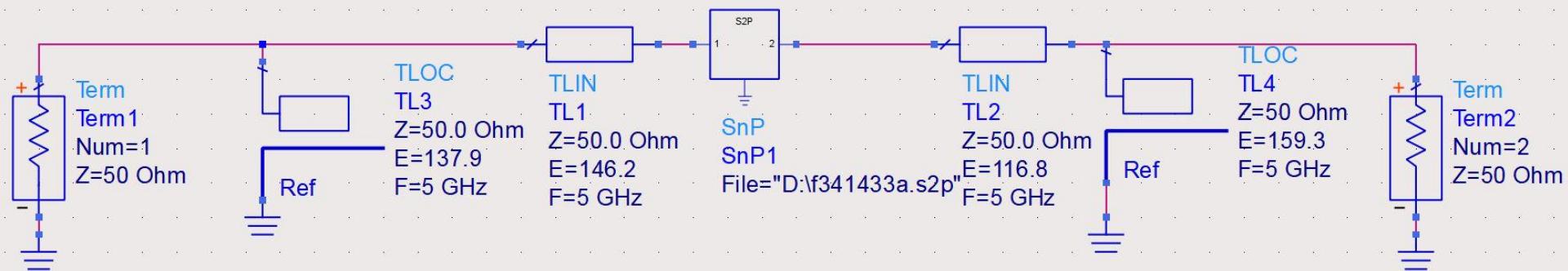
$$G_T[\text{dB}] = 1 \text{ dB} + 10 \text{ dB} + 0.2 \text{ dB} = 11.2 \text{ dB}$$

- We estimate a noise factor well below 0.75dB (quite close to the minimum ~0.6 dB)

ADS



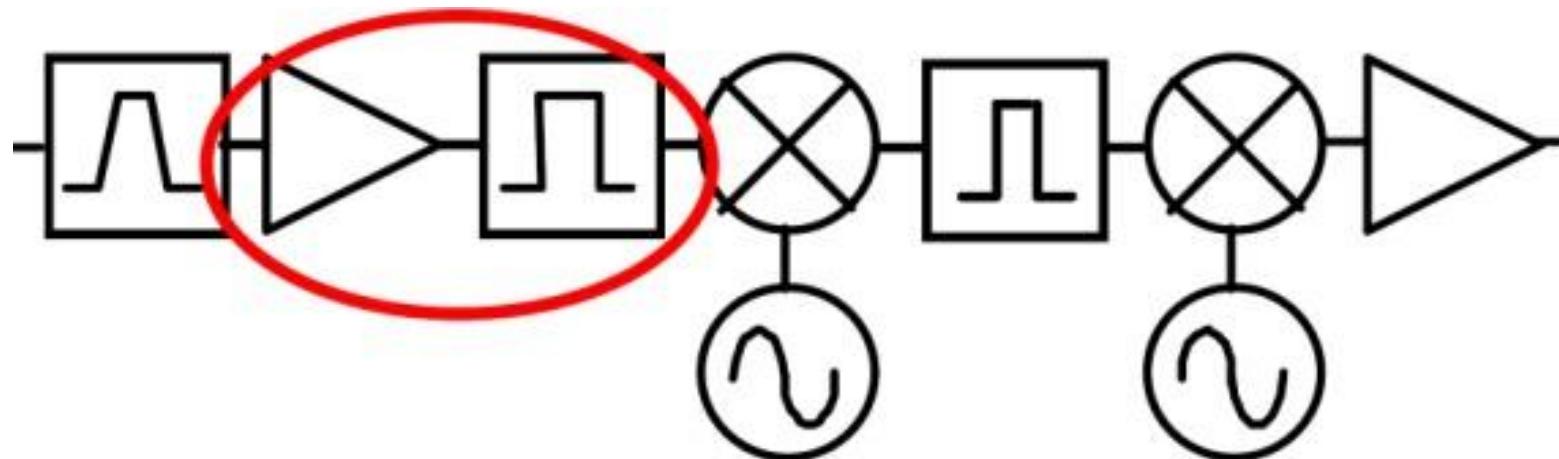
ADS



Microwave Filters

Assignment

- this structure is frequently encountered in radiocommunication systems



Microwave Filters

- Two ways of implementing filters in microwave frequency range
 - microwave specific structures (coupled lines, dielectric resonators, periodic structures)
 - **filter synthesis** with lumped elements followed by implementation with transmission lines
- the first strategy leads to more efficient filters but:
 - has lower generality
 - design is often difficult (lack of analytical relationships)

Filter synthesis

- Filter is designed with lumped elements (L/C) followed by implementation with distributed elements (transmission lines)
 - general
 - analytical relationships easy to implement on the computer
 - efficient
- The preferred procedure is **insertion loss method**

Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
 - in the passband (pass all frequencies)
 - in the stopband (reject all frequencies)

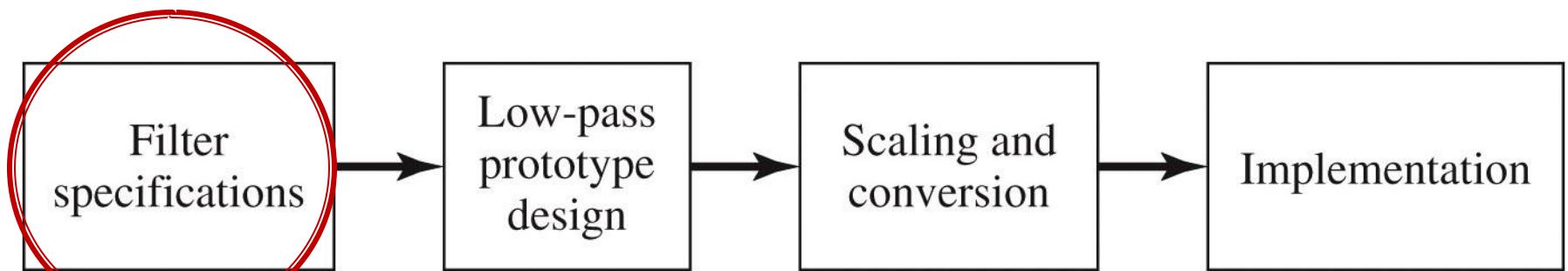
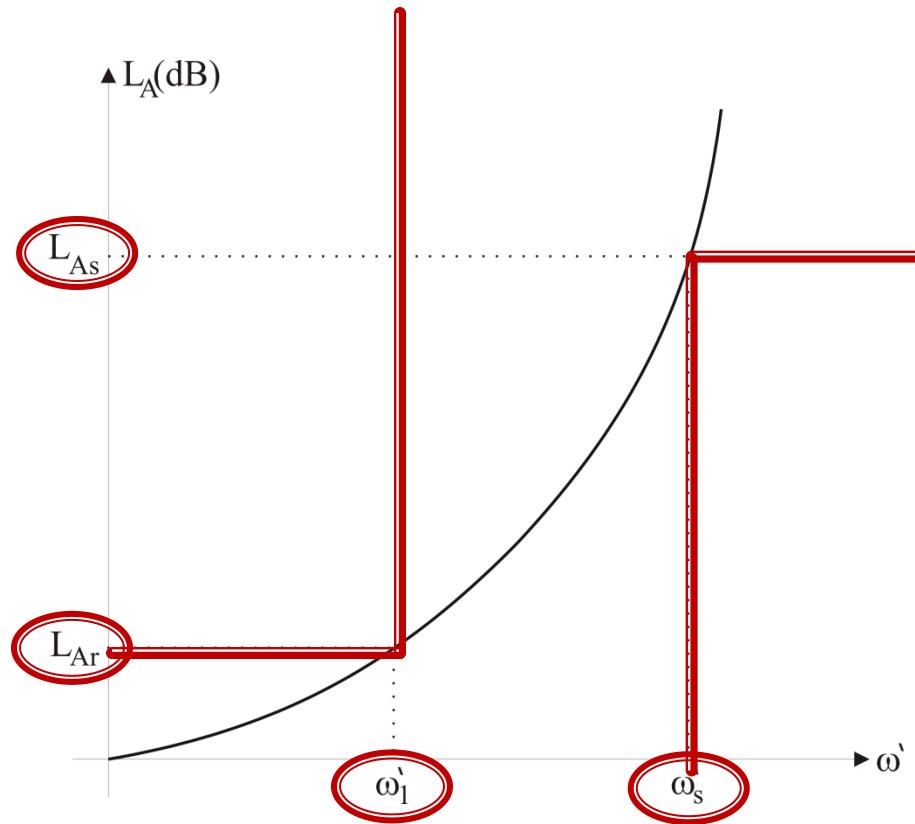


Figure 8.23

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Filter specifications

- Attenuation
 - in passband
 - in stopband
 - most often in **dB**
- Frequency range
 - passband
 - stopband
 - cutoff frequency ω_1'
usually normalized
(= 1)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
 - low-pass, high-pass, bandpass, or bandstop

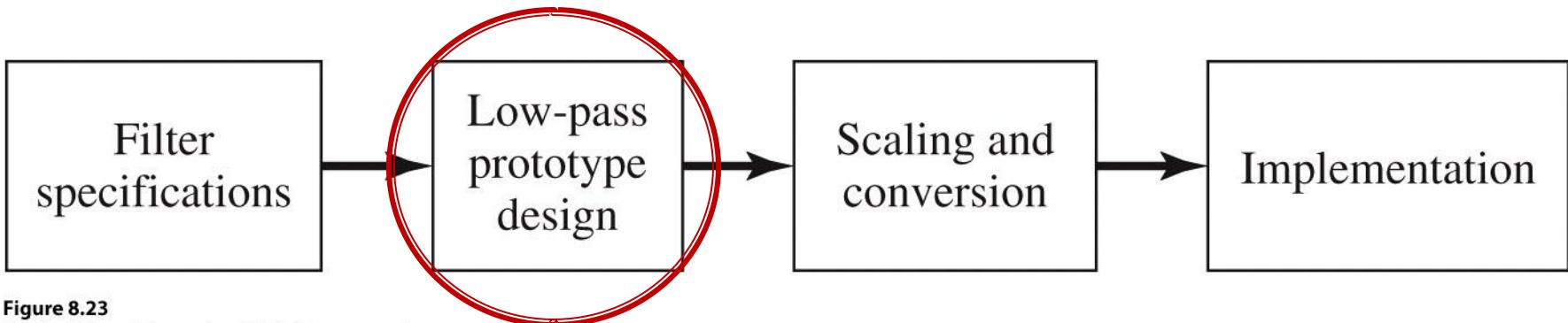


Figure 8.23

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Practical low-pass prototypes responses

- **Maximally flat filters** (Butterworth, binomial): provide the flattest possible passband response
- **Equal ripple filters** (Chebyshev): provide a sharper cutoff but the passband response will have ripples
- **Elliptic function filters**, they have equal-ripple responses in the passband as well as in the stopband,
- **Linear phase filters**, offer linear phase response in the passband to avoid signal distortion (important in some applications)

Maximally Flat/Equal ripple LPF Prototype

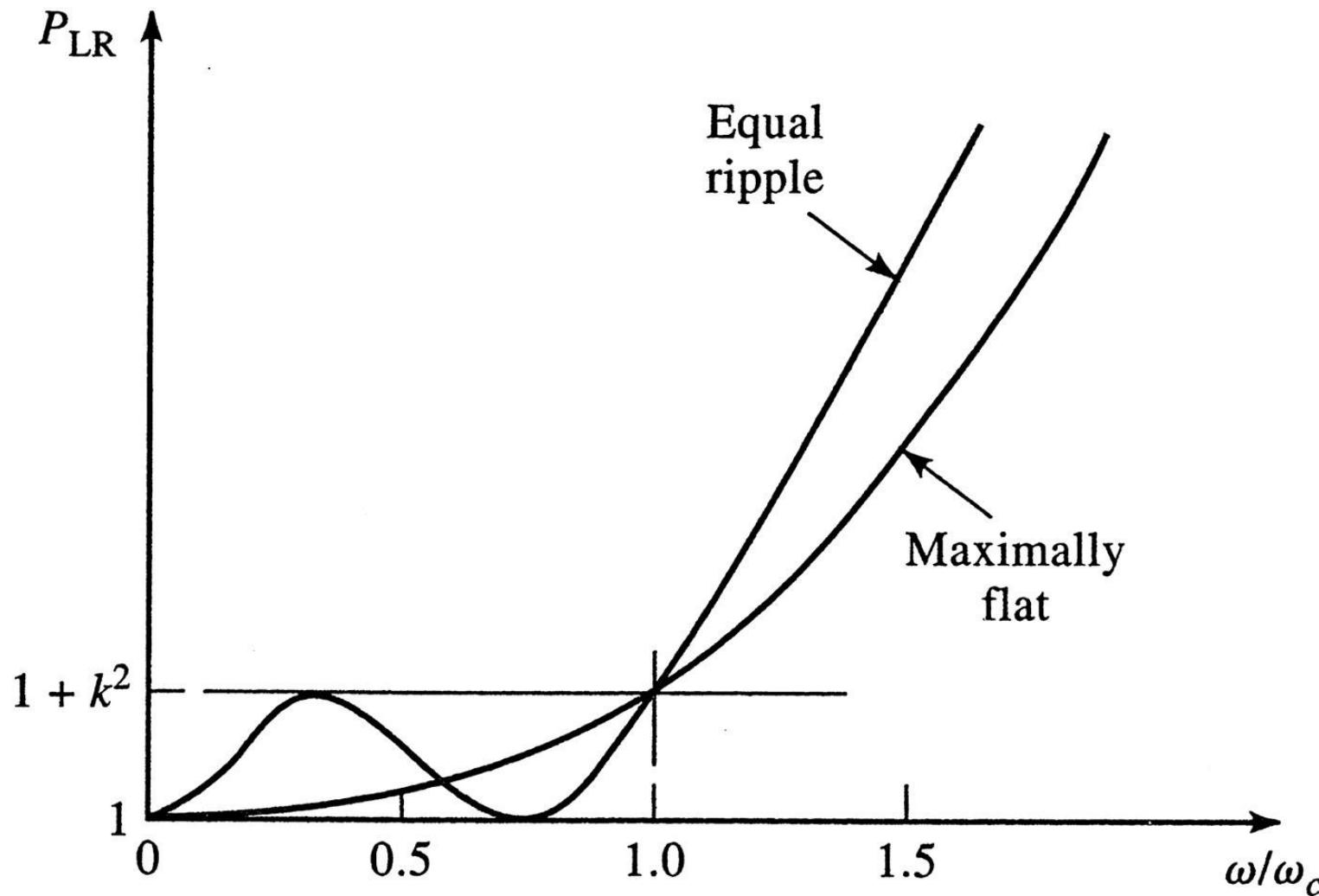


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Elliptic function LPF Prototype

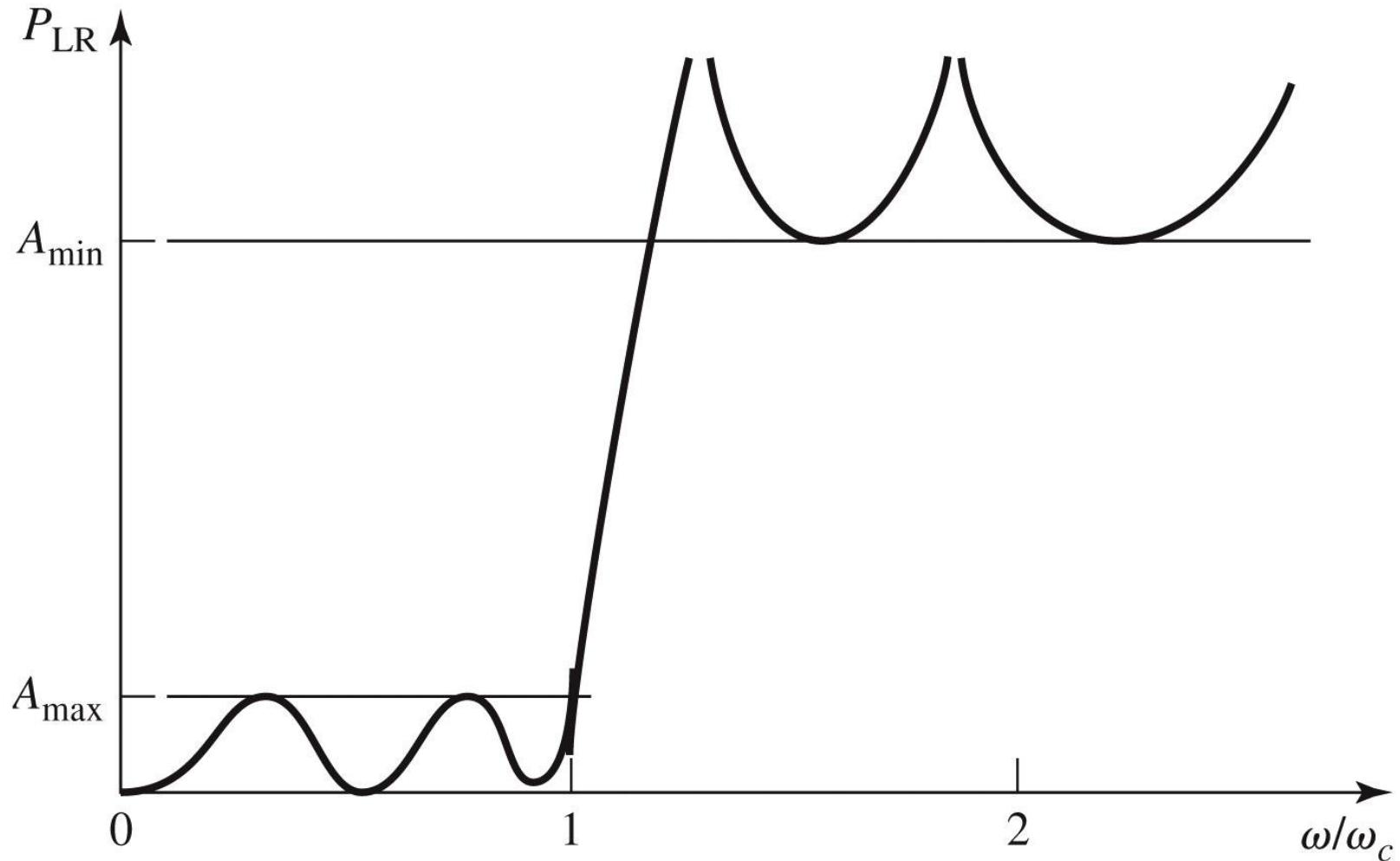


Figure 8.22

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Maximally Flat LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot \left(\frac{\omega}{\omega_c} \right)^{2N}$$

- For $\omega \gg \omega_c$

$$P_{LR} \approx k^2 \cdot (\omega/\omega_c)^{2N}$$

- attenuation increases at a rate of $20 \cdot N$ dB/decade
- k gives the attenuation at cutoff frequency (3dB cutoff imposes $k = 1$)

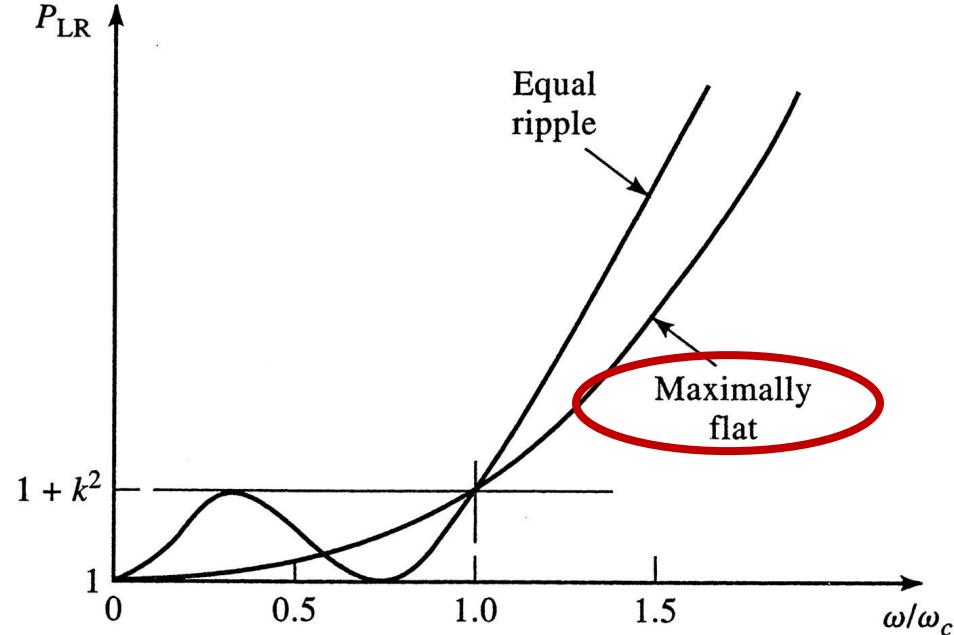


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Equal Ripple LPF Prototype

- Polynomial

$$P_{LR} = 1 + k^2 \cdot T_N^2\left(\frac{\omega}{\omega_c}\right)$$

- For $\omega \gg \omega_c$

$$P_{LR} \approx \frac{k^2}{4} \cdot \left(\frac{2 \cdot \omega}{\omega_c}\right)^{2N}$$

- attenuation increases at a rate of $20 \cdot N$ dB/decade (**also**)
- attenuation is $(2^{2N})/4$ greater than the binomial response at any given frequency where $\omega \gg \omega_c$
- the passband ripples: $1 + k^2$, k gives the ripple

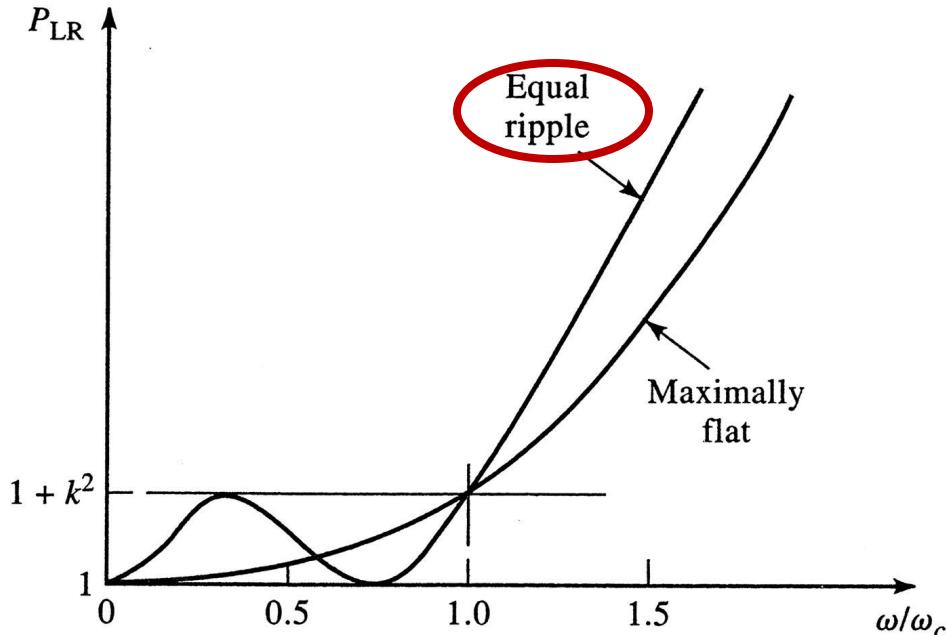
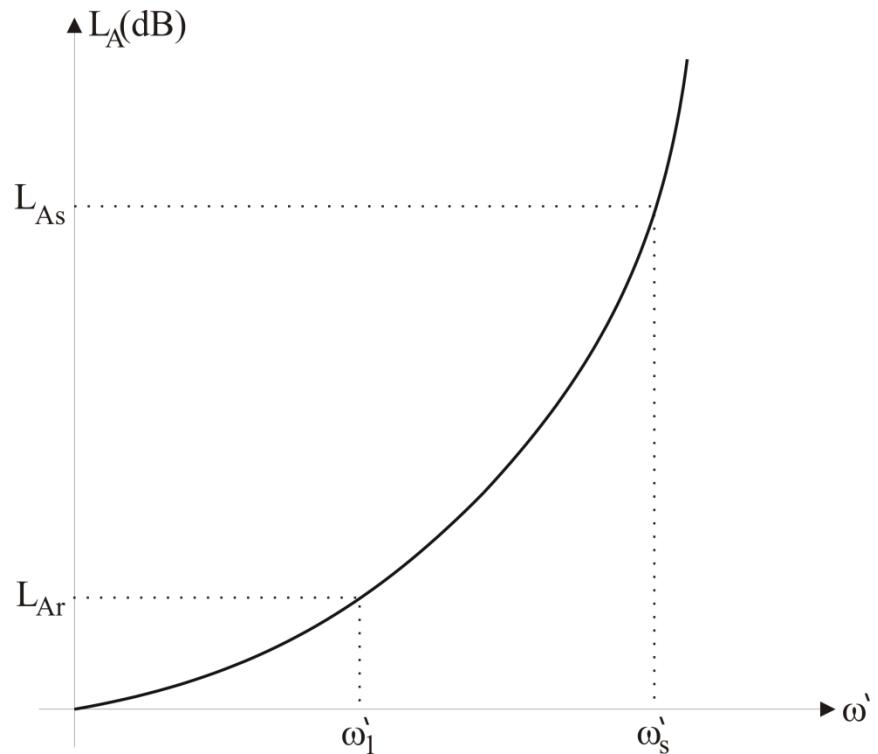


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Order (N) of the Maximally Flat filter

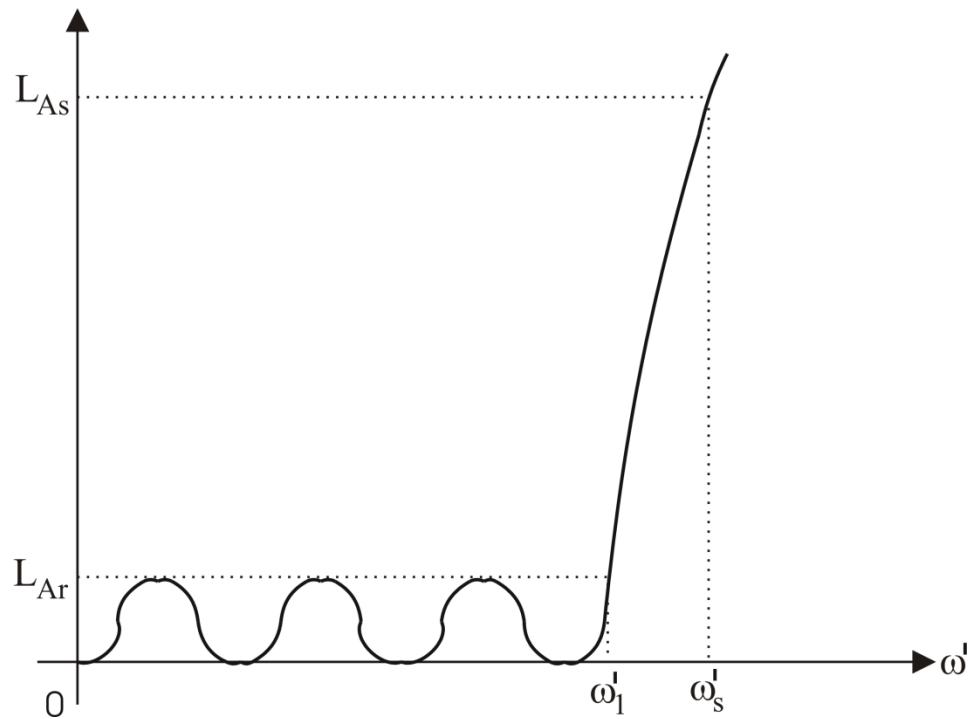
$$n \geq \frac{\log \left(\frac{10^{\frac{L_{As}}{10}} - 1}{10^{\frac{L_{Ar}}{10}} - 1} \right)}{2 \cdot \log \frac{\omega'_s}{\omega'_1}}$$

- !attenuations in **dB**



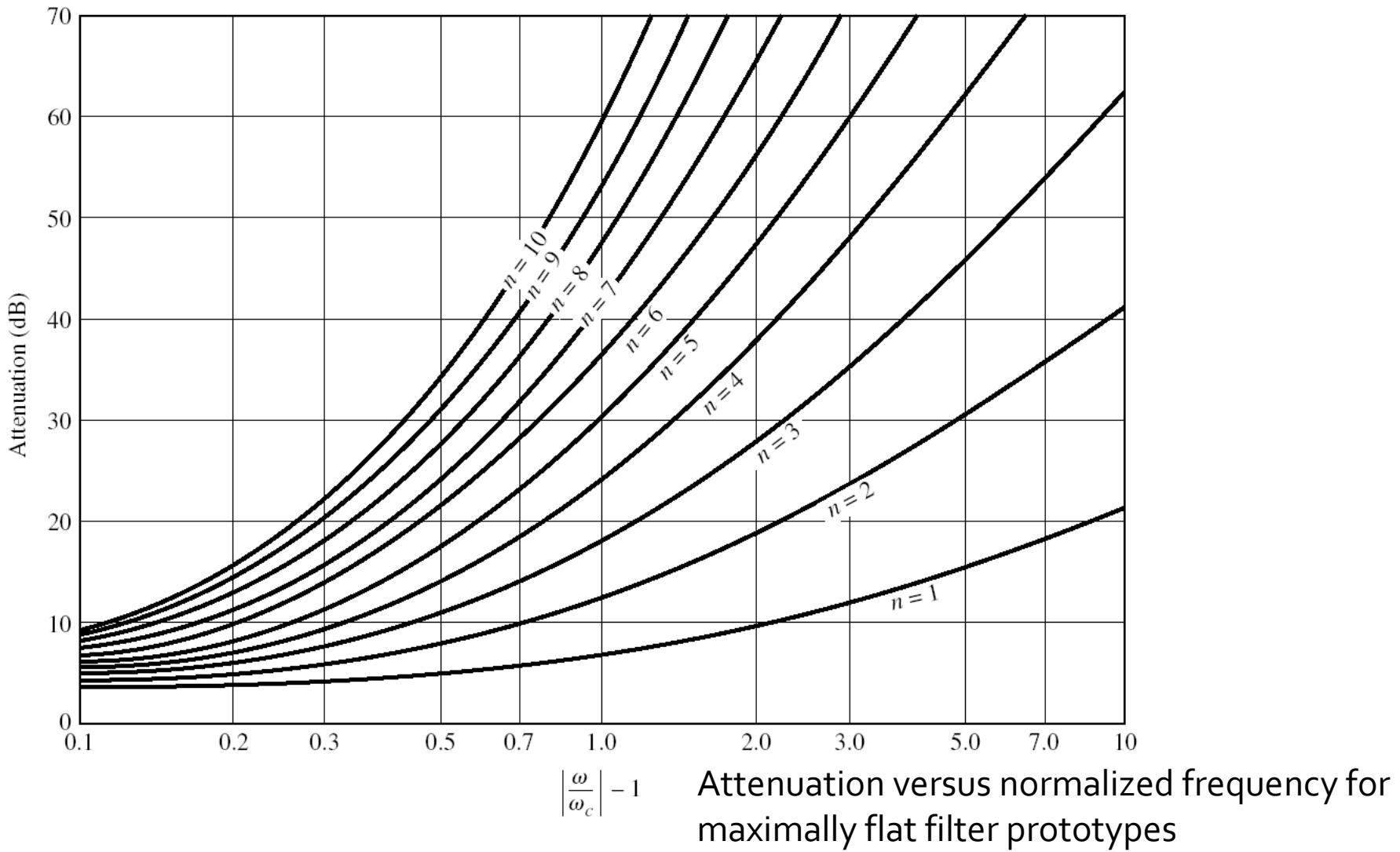
Order (N) of the Equal Ripple filter

$$n \geq \frac{\cosh^{-1} \left(\sqrt{\frac{10^{\frac{L_{As}}{10}} - 1}{10^{\frac{L_{Ar}}{10}} - 1}} \right)}{\cosh^{-1} \left(\frac{\omega'_s}{\omega'_1} \right)}$$

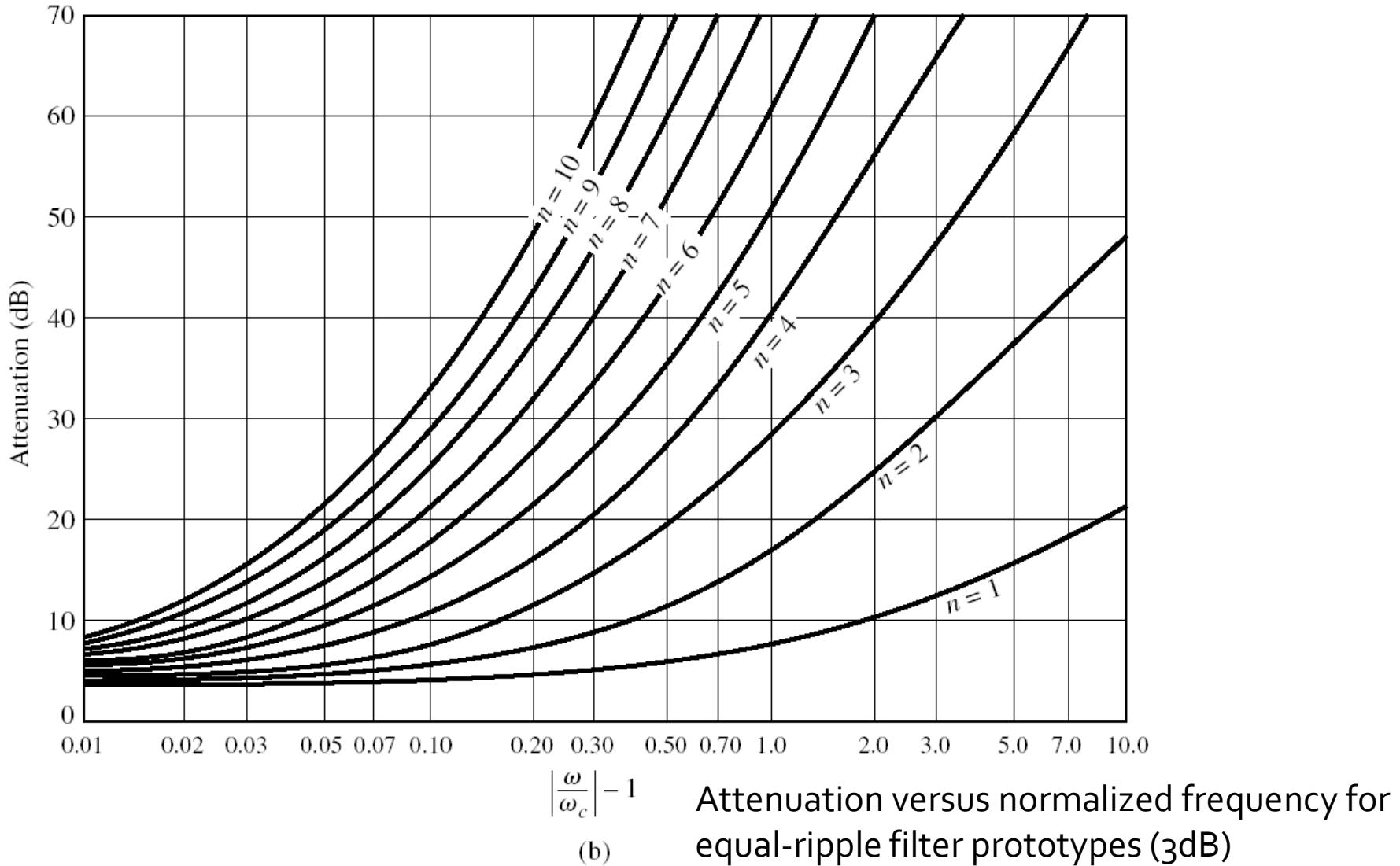


- !attenuations in **dB**

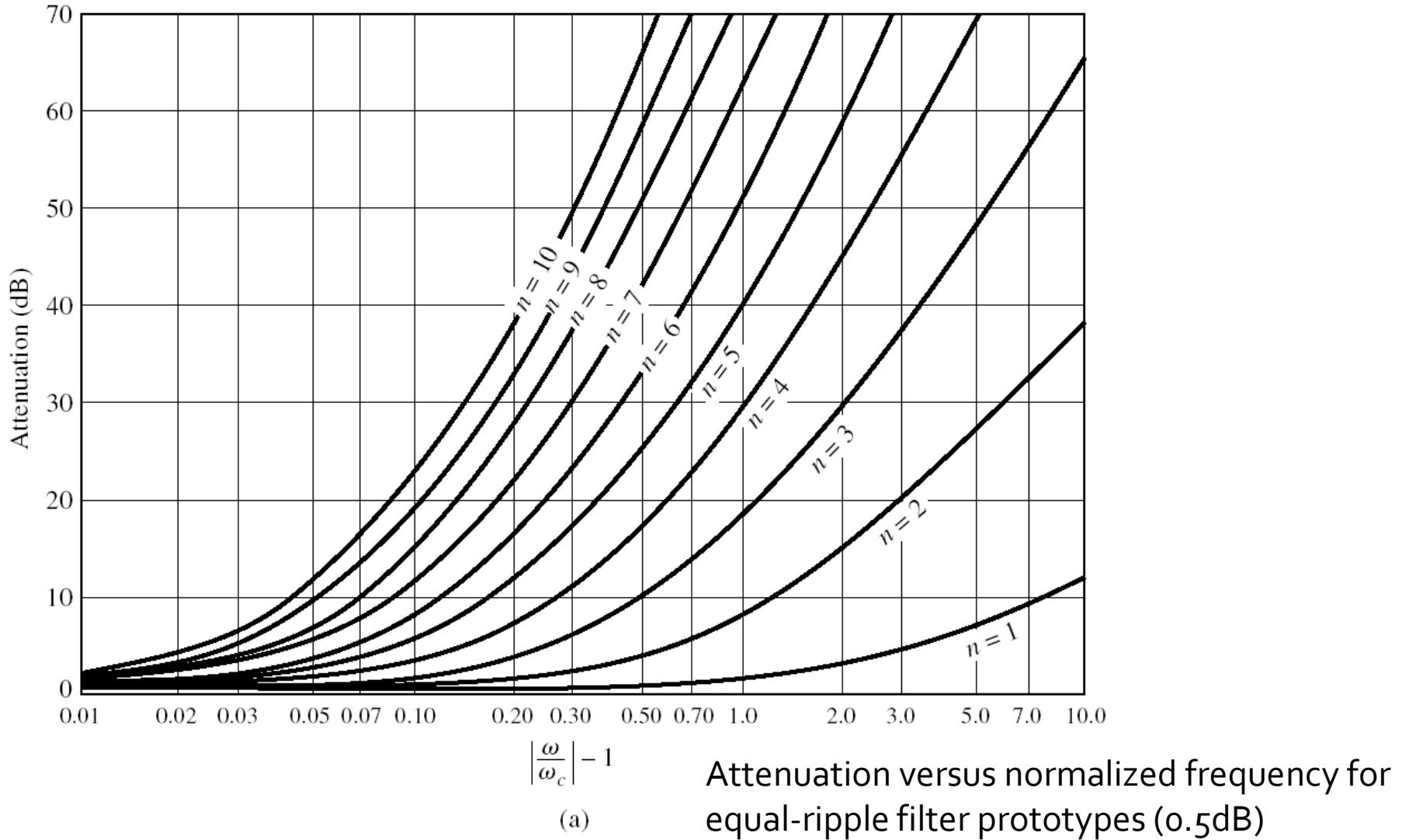
Maximally flat filter prototypes



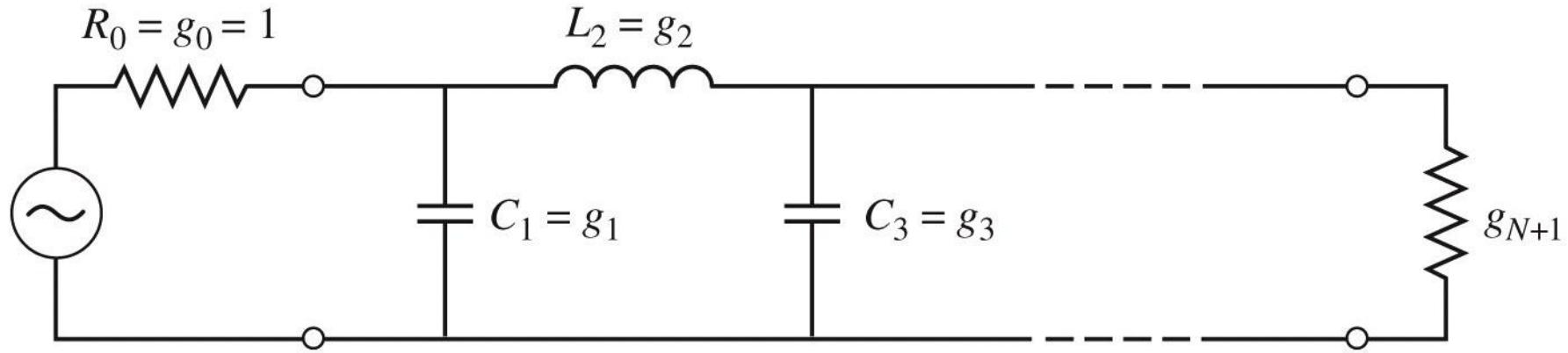
3 dB Equal-ripple filter prototypes



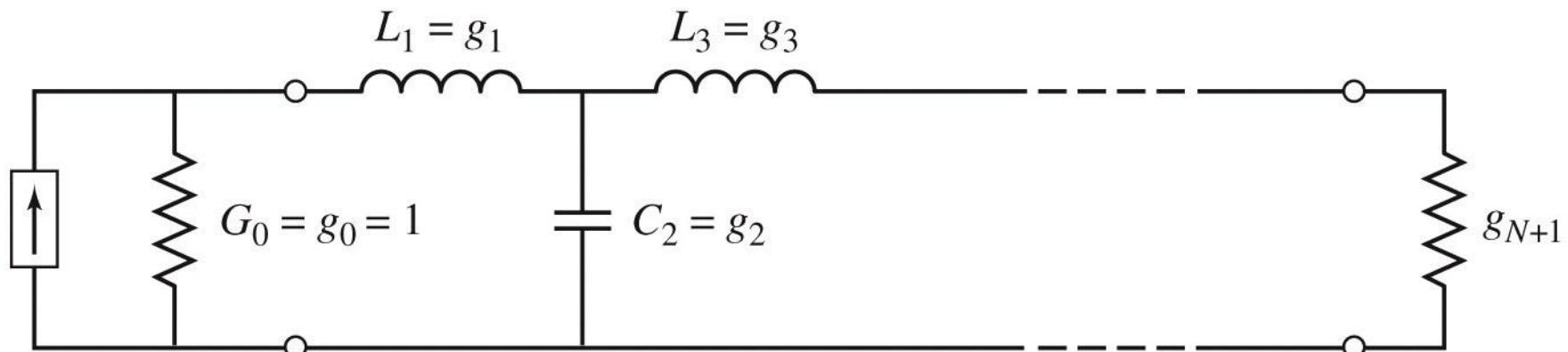
0.5 dB Equal-ripple filter prototypes



Prototype Filters



(a)



(b)

Prototype Filters

- Prototype filters are:
 - Low-Pass Filters (LPF)
 - cutoff frequency $\omega_0 = 1 \text{ rad/s}$ ($f_0 = 0.159 \text{ Hz}$)
 - connected to a source with $R = 1\Omega$
- The number of reactive elements (L/C) is the order of the filter (N)
- Reactive elements are alternated: series L / shunt C
- There are two prototypes with the same response, a prototype beginning with a shunt C element, and a prototype beginning with a series L element

Prototype Filters

- We define filter parameters g_i , $i=0, N+1$
- g_i are the element values in the prototype filter

$$g_0 = \begin{cases} \text{generator resistance } R'_0 \text{ if } g_1 = C'_1 \\ \text{generator conductance } G'_0 \text{ if } g_1 = L'_1 \end{cases}$$

$$g_k \Big|_{k=\overline{1,N}} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance } R'_{N+1} \text{ if } g_N = C'_N \\ \text{load conductance } G'_{N+1} \text{ if } g_N = L'_N \end{cases}$$

Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N} \right] , \quad k = 1, N$$

$$g_{N+1} = 1$$

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Equal-ripple LPF Prototype

- Formulas for filter parameters (iterative)

$$a_k = \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] , \quad k = 1, N \quad \beta = \ln\left(\coth\frac{L_{Ar}}{17.37}\right)$$

$$\gamma = \sinh\left(\frac{\beta}{2 \cdot N}\right) \quad b_k = \gamma^2 + \sin^2\left(\frac{k \cdot \pi}{N}\right) , \quad k = 1, N$$

$$g_1 = \frac{2 \cdot a_1}{\gamma}$$

$$g_k = \frac{4 \cdot a_{k-1} \cdot a_k}{b_{k-1} \cdot g_{k-1}} , \quad k = 2, N$$

$$g_{N+1} = \begin{cases} 1 & \text{for odd } N \\ \coth^2\left(\frac{\beta}{4}\right) & \text{for even } N \end{cases}$$

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

N	0.5 dB Ripple										
	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841
3.0 dB Ripple											
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

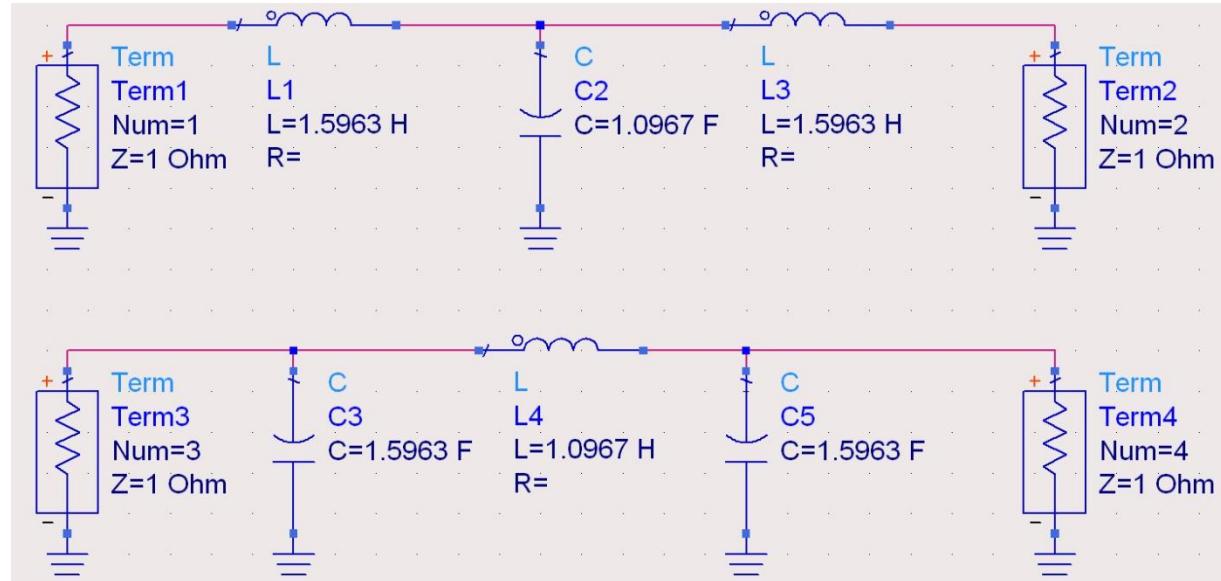
- For even N order of the filter ($N = 2, 4, 6, 8 \dots$) equal-ripple filters **must** closed by a load impedance $g_{N+1} \neq 1$
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial ...) to $g_L = 1$

Example

- Design a **3rd order** bandpass filter with **0.5 dB ripples** in passband. The center frequency of the filter should be **1 GHz**. The fractional bandwidth of the passband should be **10%**, and the impedance **50Ω** .

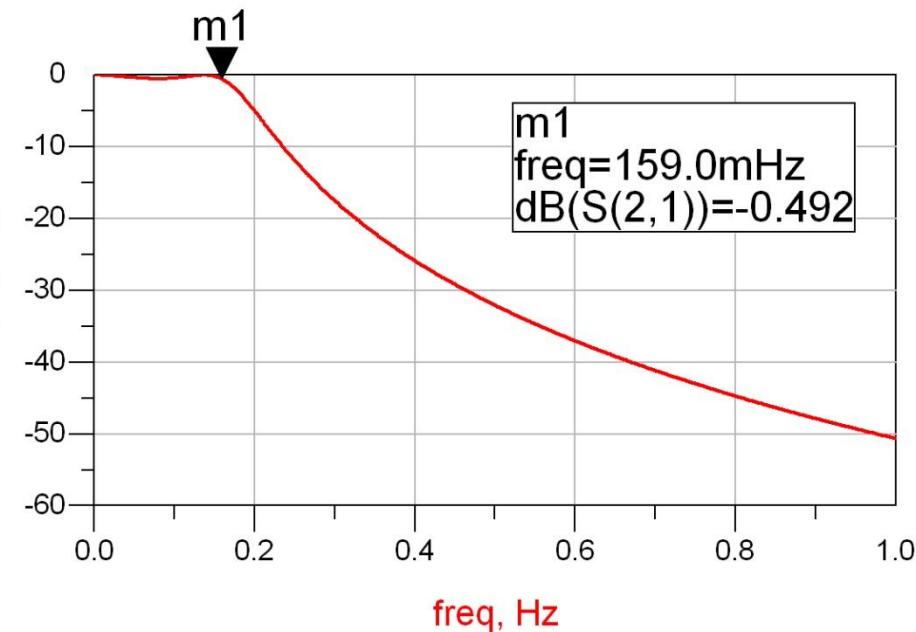
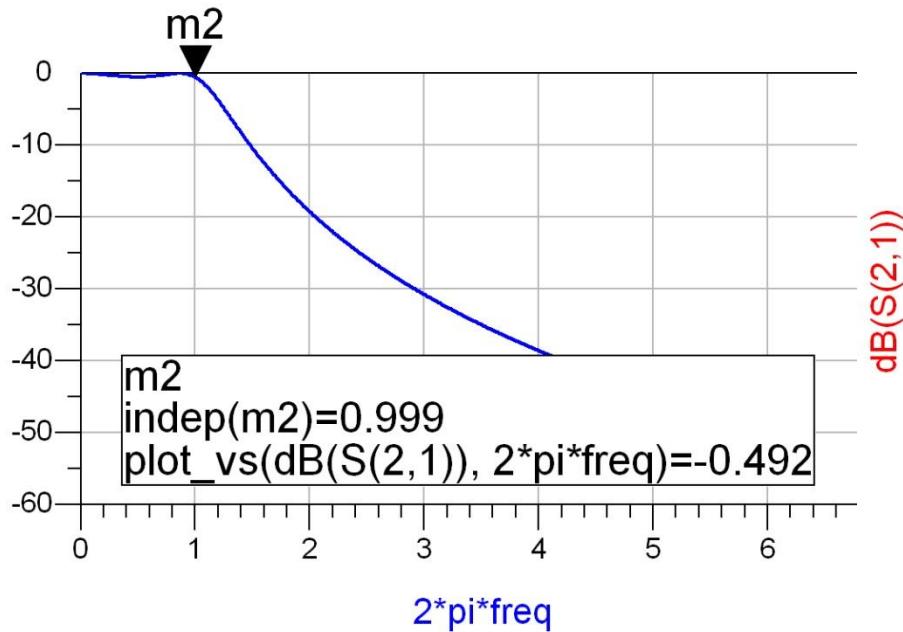
LPF Prototype

- 0.5dB equal-ripple table or design formulas:
 - $g_1 = 1.5963 = L_1/C_3$,
 - $g_2 = 1.0967 = C_2/L_4$,
 - $g_3 = 1.5963 = L_3/C_5$,
 - $g_4=1.000 = R_L$



LPF Prototype

- $\omega_o = 1 \text{ rad/s}$ ($f_o = \omega_o / 2\pi = 0.159 \text{ Hz}$)



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